Abstract: In current practice and existing solutions, temperature control in commercial buildings is mostly done independently of occupant feedback. Typically, an acceptable temperature range for the occupancy level is estimated, and the control solution is designed to maintain temperature within that range during occupancy hours while optimizing energy usage at the same time. In this work we incorporate active user (occupant) feedback to minimize aggregate user discomfort. User feedback is accepted in a convenient binary format which is then used to estimate the user’s comfort range (discomfort function), taking into account possible inaccuracies in the feedback provided. The control algorithm design also takes the energy cost into account, trading it off optimally with the aggregate user discomfort. A lumped heat transfer model based on thermal resistance and capacitance is used to model a multi-zone building, and singular perturbation theory is utilized to analyze the system. Under sufficient time scale separation between temperature dynamics and user feedback frequency, we establish convergence of the proposed solution to the desired temperature that minimizes the sum of the energy cost plus user discomfort. Simulation results on a four-room example are presented to demonstrate the performance of the proposed approach and validate the model.

Keywords: Temperature control, singular perturbation method, occupant comfort feedback.
schools, libraries, offices etc. which have a very diverse collection of occupants. A diverse group of occupants will have different ranges of comfort as well as different tolerance levels towards temperature variations beyond their individual comfort ranges. Since the location of the occupants in a building is not static, and even their comfort preferences could change over time, a temperature control system that is based on a-priori knowledge of occupant location and preferred temperature range is not desirable. In other words, the system needs to learn these dynamically through user (occupant) feedback and incorporate it into the control strategy. Therefore, achieving energy efficiency in a building taking into account the comfort levels of its occupants needs a new and unique approach towards problem solution.

In this paper we present a control solution to optimally tradeoff the overall energy cost with the aggregate occupant discomfort. A novel algorithm is developed and analyzed using gradient optimization and singular perturbation theory. We use simple feedback from users in the form of ‘heat up’ or ‘cool down’, which are further consolidated to estimate their comfort ranges or the discomfort functions, taking into account possible errors in the users’ feedback close to the boundaries of their comfort range. The user input thus obtained when combined with the resulting energy cost, determines the direction in which the energy control input is adjusted. We consider a multi-zone building, and use a lumped heat transfer model based on thermal resistance and capacitance for system analysis. Singular perturbation theory is used to analyze the system, with temperature evolution on a faster time scale and user input on a relatively slower time scale. With this time scale separation, the proposed control algorithm achieves convergence to a desired temperature that minimizes the sum of total energy cost and the aggregate user discomfort. Simulation results are presented for a four-room model to demonstrate the performance and effectiveness of the proposed algorithm. It is worth noting that our work is loosely related to Thermovote [Erickson and Cerpa (2012)] and a more recent work [Purdon et al. (2013)] that seek user feedback at binary/multiple levels, and determine the direction of temperature adjustment based on the average user vote. However, unlike these existing studies, we pose the problem formally as an optimization question that takes into account the temperature dynamics as a function of the energy input. Furthermore, while we prove that our control solution converges to the optimal tradeoff point between energy cost and user discomfort, no such guarantees are known to hold for the existing approaches.

2. SYSTEM MODEL AND CONTROL ALGORITHM

We first present a model for the building infrastructure and obtain the equations governing the system, and then pose the temperature control problem as a convex optimization question. We then describe a control law whose stability and optimality is established in Section 3.

2.1 Problem Formulation

The first major step towards designing a building energy control system is determining the choice of the building heat transfer model. Different models towards this purpose have been proposed in the literature, which include the finite element method based model [Meebee (2011)], lumped mass and energy transfer model [Riederer et al. (2002), Wu et al. (2008)], and graph theoretic model based on electrical circuit analogy [Boyer et al. (1996), Fraisse et al. (2002), Xu et al. (2008), Athienitis et al. (1985)]. The system model selection entails a tradeoff between computational efficiency and accuracy of representation of the temperature dynamics. The electrical analogy approach to modeling multiple interconnected zones reduces the heat transfer model to an equivalent electrical circuit network. The model can be further modified to include building occupancy, room and heating equipment dynamics [Athienitis et al. (1985), Chandan (2010)]. In this paper we take this electrical circuit analogy approach, and combine it with occupant discomfort feedback modeling.

A building is modeled as a collection of interconnected zones, with energy/temperature dynamics evolving according to a lumped heat transfer model. In the lumped heat transfer model, a single zone is modeled as a thermal capacitor and a wall is modeled as an RC network. This results in the standard lumped 3R2C wall model [Fraisse et al. (2002)]. The heat flow modeling is based on temperature difference and thermal resistance: \( Q = \Delta T/R \), where \( \Delta T \) is the temperature difference, \( R \) is the thermal resistance and \( Q \) is the heat transferred across the resistance. This is analogous to the current due to voltage difference across a resistor. Also, note that the thermal capacitance denotes the ability of a space to store heat: \( C \frac{dT}{dt} = Q \).

The heat flow and thermal capacitance model can be written for all the thermal capacitors in the system, with \( T_i \) as the temperature of the ith capacitor. Consider the system to have \( n \) thermal capacitors and \( l \) thermal resistors. With additional sources of heat input such as ambient environment, we can write the overall heat transfer model of the system with \( m \) zones as [Mukherjee et al. (2012)]:

\[
C\dot{T} = -DR^{-1}D^TT + B_0T_\infty + Bu + B_w,
\]

where \( T \in \mathbb{R}^n \) is the temperature vector (representing the temperature of the thermal capacitors in the 3R2C model), \( u \in \mathbb{R}^m \) is the vector of heat inputs into the different zones of the building, and \( B \in \mathbb{R}^{n \times m} \) is the corresponding input matrix. Also, note that \((T,u)\) are functions of time \((T(t),u(t))\) and accordingly \( T = \frac{dT}{dt} \). Note that positive values of \( u \) correspond to heating the system while negative values of \( u \) correspond to cooling. In the above equation, \( C \in \mathbb{R}^{n \times n} \) consists of the wall capacitances and is a diagonal positive definite matrix; \( R \in \mathbb{R}^{l \times l} \) consists of the thermal resistors in the system and is a diagonal positive definite matrix as well. Also, \( D \in \mathbb{R}^{l \times l} \) is the incidence matrix, mapping the system capacitances to the resistors, and is of full row rank [Lombard et al. (2008)], and \( B_0 = -DR^{-1}d^T_0 \in \mathbb{R}^n \) is a column vector with non-zero elements denoting the thermal conductances of nodes connected to the ambient. Further, \( T_\infty \) is the ambient temperature, and \( w \in \mathbb{R}^m \) is the thermal noise in the different zones. We represent the snapshot of the above parameters in Table 1 below for quick reference.

In this study we neglect the thermal noise, and so our model equation (1) becomes:

\[
C\dot{T} = -DR^{-1}D^TT + B_0T_\infty + Bu.
\]
In our model, the zones are picked such that each of them has a heating/cooling unit, which in turn implies that \( B \) is of full row rank. Also, since matrix \( D \) is of full row rank the product \( DR^{-1}D' \) is a positive definite matrix. The vector of zone temperatures, denoted by \( y \) (which is a function of \( T \)) can be expressed as,
\[
y = B^T T. \tag{3}
\]

Our overall minimization objective (overall cost) is the sum of two terms: (i) energy cost (i.e., cost of heating/cooling), and (ii) aggregate discomfort cost of the occupants. The energy cost (i) is expressed as \( \frac{1}{2} u^T \Gamma u \), where \( \Gamma \) is a positive definite matrix. Note that the energy cost is quadratic in the heat input vector \( u \) [Mukherjee et al. (2012)], and is consistent with other works such as [Kelman et al. (2011)] which approximates energy cost as a quadratic function of the mass flow rate. Furthermore, our framework and analysis also extends to other convex energy costs. Let \( S_j \) denote the set of all occupants in zone \( j \), and \( \rho = \sum_{j=1}^{\infty} |S_j| \) be the total number of occupants in the building. Also let \( G_s \) denote the (convex) discomfort function of occupant \( s \) in zone \( j \). Then the aggregate occupant discomfort cost (ii) is expressed as \( \sum_{j=1}^{\infty} \sum_{s \in S_j} G_s(y_j(T)) \), where \( y_j(T) = [B^T T_j] \) from (3) denotes the \( j \)th element of \( y \), or the temperature of zone \( j \). Our minimization objective is thus expressed as,
\[
U(u, T) = \frac{1}{2} u^T \Gamma u + \gamma \sum_{j=1}^{\infty} \sum_{s \in S_j} G_s(y_j(T)). \tag{4}
\]

In (4), \( \gamma \) is a scalar constant that defines the relative weight provided to the aggregate occupant discomfort, as compared to the energy cost. Next we obtain a control strategy that can guide the control input \( u \) so that it minimizes the total system cost as defined in (4) subject to the temperature dynamics (2) and (3).

2.2 Solution Approach

Assuming a constant ambient temperature \( T_\infty \), and using equilibrium condition (setting \( \dot{T} = 0 \) in (2)) we obtain:
\[
T = h(u) = (DR^{-1}D')^{-1}(B_0 T_\infty + B u). \tag{5}
\]

Define, \( J(u) = U(u, h(u)) \), \( i.e., \) \( J(u) \) is obtained by plugging in \( T = h(u) \) (from (5)) into (4). Note that energy cost term in (4) is strictly convex in \( u \); and the aggregate occupant term is convex in \( T \), and therefore convex in \( u \) when \( T \) is set to \( h(u) \), since \( h(u) \) is affine in \( u \). This implies that \( J(u) \) is strictly convex in \( u \). Therefore \( J(u) \) has a unique optimal solution \( u^* \). Define
\[
T^* = h(u^*), \tag{7}
\]
which is also unique by definition.

With the goal of driving the system to \((u^*, T^*)\), we propose the control input \( u \) be updated once every \( \Delta \) time units as
\[
u_{k+1} = u_k - \eta (\Gamma u + \gamma YAF(y)), \tag{8}
\]
where \( \eta \) is a scalar that can be loosely interpreted as the “feedback gain” of the system. Furthermore, \( Y \in \mathbb{R}^{m \times m} \) in the above is the Jacobian obtained using (3) and the equilibrium condition (5), expressed as
\[
Y = \frac{\partial y}{\partial u} = B^T (DR^{-1}D')^{-1} B. \tag{9}
\]

Also, \( A \in \mathbb{R}^{m \times \rho} \) is the zone-occupant matrix that indicates which occupants are present in a zone (\( A_{js} = 1 \) if \( s \in S_j \), and 0 otherwise), and \( F(y) \in \mathbb{R}^{\rho \times 1} \) is the “marginal discomfort” vector of the occupants, obtained by taking partial derivative of the occupant discomfort functions with respect to \( y \). In other words, the \( s \)th element of \( F(y) \), where \( s \in S_j \), is obtained as
\[
F_s(y_j) = \frac{dG_s(y_j)}{dy_j}, \quad s \in S_j. \tag{10}
\]

Comparing (8) with (4) provides the motivation of our control algorithm: roughly speaking, (8) updates \( u \) in the gradient direction of \( U(u, T) \), while taking in account the relationship between \( T \) and \( u \) at equilibrium, as given by (5). In other words, it attempts to update \( u \) is the direction of \( -\nabla J(u) \), where \( J(u) \) is defined by (6). In this interpretation, \( \eta \) represents the constant “step size” associated with the gradient descent.

Note however that using (4) - (6), \( \nabla J(u) \) is expressed as:
\[
\nabla J(u) = \Gamma u + \gamma Y \Lambda F(B^T h(u)). \tag{11}
\]

From (11) we note that update of \( u \) in the gradient direction of \( J(u) \) requires user discomfort feedback at \( y = B^T h(u) \), the equilibrated zone temperatures corresponding to \( u \). In practice, however, a user \( s \in S_j \) will provide a comfort feedback at the current temperature it experiences, \( y_j = [B^T T_j] \) (different in general from the equilibrated temperature \( [B^T h(u)]_j \)), which is what we incorporate into our control algorithm as stated in (8). This implies that our control algorithm as described in (8) does not exactly move \( u \) in the gradient direction \( (-\nabla J(u)) \). The effect of this difference (error) can be analyzed using singular perturbation theory [Khalil (2002), Kokotovic et al. (1986)], which in our case requires (for convergence to optimality) that the occupant feedback be collected after long intervals (i.e. \( \Delta \) is large), allowing the temperature \( T \) to settle down close to \( h(u) \) before the next occupant feedback collection.

Towards developing a singular perturbation model of our system, we first consider a continuous approximation to the evolution of the control input \( u \):
\[
\dot{u} \approx \frac{u_{k+1} - u_k}{\Delta} = -\frac{\eta}{\Delta} (\Gamma u + \gamma YAF(y)). \tag{12}
\]

Note that time step \( \Delta \) is the interval at which user feedback is solicited and the control input \( u \) is updated. A larger \( \Delta \) implies a slower evolution of \( u \). We next express the system evolution in the time scale of the evolution of \( u \) (slower time scale as compared to the time scale at which \( T \) evolves). Define \( \tau = \frac{t}{\Delta} \) as the perturbation parameter; then \( \tau = \frac{t}{\Delta} \) is the slower time scale. Then

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T \in \mathbb{R}^n )</td>
<td>Temperature vector</td>
</tr>
<tr>
<td>( u \in \mathbb{R}^m )</td>
<td>Heat input vector</td>
</tr>
<tr>
<td>( C \in \mathbb{R}^{m \times n} )</td>
<td>Wall capacitances</td>
</tr>
<tr>
<td>( R \in \mathbb{R}^{n \times n} )</td>
<td>Thermal resistors in the system</td>
</tr>
<tr>
<td>( D \in \mathbb{R}^{n \times n} )</td>
<td>Incidence matrix</td>
</tr>
<tr>
<td>( B_0 \in \mathbb{R}^n )</td>
<td>Thermal conductances of nodes connected to the ambient</td>
</tr>
<tr>
<td>( T_\infty \in \mathbb{R}^n )</td>
<td>Ambient temperature</td>
</tr>
<tr>
<td>( w \in \mathbb{R}^m )</td>
<td>Thermal noise</td>
</tr>
<tr>
<td>( B \in \mathbb{R}^{m \times m} )</td>
<td>Input matrix</td>
</tr>
</tbody>
</table>
\[
\frac{dT}{dt} = \kappa(T - h(u))T + B_0T_{\infty} + Bu.
\] 
(Equations (14) and (15) represent a singularly perturbed system. Note, \(\Delta \uparrow \Rightarrow \epsilon \downarrow\) leading to steady state condition for temperature evolution. In the next section as we establish the global asymptotic stability of our system as given by equations (14) and (15).

Finally, note that implementation of our control algorithm would require that \(F_s(y_j)\), the "marginal discomfort" value of user \(s\) in zone \(j\) at the current zonal temperature \(y_j = [B^T T]_j\), be reasonably estimated from the discomfort feedback of \(s\) at any time. In practice, the occupants may provide the feedback in some simple form describing their actual level of discomfort ("I am feeling hot", "I am feeling very cold", etc.). This feedback must be processed to estimate the marginal discomfort (derivative of the actual discomfort function), as we do in our simulation study described in Section 4.

### 3. STABILITY ANALYSIS

The system evolution is governed by the set of equations (14) and (15). In equation (15) the coefficient \(DR^{-1}D^T\) is positive definite which makes the unforced system (with \(u = 0\)) exponentially stable. We use singular perturbation analysis [Kokotovic et al. (1986)] to establish the condition for stability of the system.

Next we introduce Lyapunov functions \(V(u)\) and \(W(u, T)\) that will be used in our stability analysis:

\[
V(u) = J(u) - J(u^*), \quad \text{and} \quad W(u, T) = (T - h(u))^T P(T - h(u)),
\]

where \(P\) in the above equation is a symmetric positive definite matrix (the exact choice of matrix \(P\) will be determined at a later stage). We now define a combined Lyapunov function \(L(u, T)\):

\[
L(u, T) = (1 - \alpha)V(u) + \alpha W(u, T),
\]

where \(\alpha\) satisfies \(0 < \alpha < 1\).

We start by evaluating the conditions to establish stability using Theorem 2.1 and Corollary 2.1 from chapter 7 of [Kokotovic et al. (1986)]. We propose the following comparison functions for the analysis:

\[
\Psi(u) = \|\nabla J(u)\|, \quad \text{and} \quad \Phi(T - h(u)) = \|T - h(u)\|.
\]

We assume that the user discomfort function \(G_s(y_j)\) has bounded second derivative i.e. there exists a \(\kappa < \infty\):

\[
\frac{d^2 G_s(y_j)}{dy_j^2} \leq \kappa, \forall y_j, s \in S_j.
\]

Note that from equation (10) above:

\[
F_s'(y_j) = \frac{dF_s(y_j)}{dy_j} = \frac{d^2 G_s(y_j)}{dy_j^2} \leq \kappa, s \in S_j.
\]

We can now use the Mean Value Theorem to assert:

\[
F_s(y_j) - F_s([B^T h(u)]_j) = F_s'(\tilde{y}_j)(y_j - [B^T h(u)]_j),
\]

for some \(\tilde{y}_j \in [y_j, [B^T h(u)]_j], s \in S_j\). (23)

Applying Cauchy-Schwartz inequality on (23), for \(s \in S_j\):

\[
\|F_s(y_j) - F_s([B^T h(u)]_j)\| \leq \|F_s'(\tilde{y}_j)\|\|y_j - [B^T h(u)]_j\|.
\]

Define \(\kappa = \tilde{\kappa}\|B\|\). Then from (24), (22) and (3), we have

\[
\|F_s(y_j) - F_s([B^T h(u)]_j)\| \leq \kappa\|T - h(u)\|.
\]

Now we evaluate the conditions in Assumption 2.3 [chapter 7 of Kokotovic et al. (1986)] on \(V(u)\) to obtain:

\[
\frac{\partial V}{\partial u} \left( \frac{dT}{dt} h(u) \right) = -\eta(T J(u))(T J(u)) \leq -\eta \Psi^2(u),
\]

and

\[
\frac{\partial V}{\partial u} \left( \frac{dT}{dt} h(u) \right) \leq -\eta\Psi(T - h(u)).
\]

The above expressions are obtained by taking partial derivative of \(V(u)\) as defined in (16) and substituting \(\frac{dT}{dt}\) from (14). Next, evaluating conditions from Assumption 2.2 [chapter 7 of Kokotovic et al. (1986)] on \(W(u, T)\) yields (28) and (29), stated below.

\[
\frac{\partial W}{\partial T} \left( \frac{dT}{dt} h(u) \right) = -2(T - h(u))^T P\frac{1}{D^T D} (D^T D)^{-1} (T - h(u)) \leq -2\lambda_{min} \Psi^2(T - h(u)).
\]

where \(\lambda_{min}\) is the minimum eigenvalue of the symmetric part of the matrix \((P^{-1} D^T D)^{-1}\), assumed to be positive definite. Note that the symmetric positive definite matrix \(P\) must be chosen such that \((P^{-1} D^T D)^{-1}\) is positive definite. One choice would be \(P = C\) since \((D^T D)^{-1}\) is positive definite. Another choice is \(P = I\); it is reasonable to assume that the symmetric part of the matrix \((C^{-1} D^T D)^{-1}\) has positive eigenvalues, as we have verified to hold for the data set used in our simulation study presented in the next section.

\[
\frac{\partial W}{\partial u} \left( \frac{dU}{dt} \right) = 2\eta(T - h(u))^T P(D^T D)^{-1} B \nabla J(u) + 2\gamma(T - h(u))^T P(D^T D)^{-1} B \nabla \Psi(T - h(u)) \leq 2\eta\|P\|(D^T D)^{-1} \|B\|\Psi(T - h(u)) + 2\gamma\|P\|(D^T D)^{-1} \|B\|\Psi(T - h(u)).
\]

Equation (28) is obtained using (15) and the definition of \(W(u, T)\) in (17). Equation (29) is obtained using (17) as well as (14), (11) and (25). Given the Lyapunov functions \(V(u)\) and \(W(u, T)\) satisfy the conditions (26) - (29) and are radially unbounded by definition, Theorem 2.1 with Corollary 2.1 of [Kokotovic et al. (1986)] states that for every \(\alpha, L(u, T)\) as given by equation (18), is a Lyapunov function for \(\epsilon < \epsilon^*\). For our system \(\epsilon^*\) can be obtained in terms of the conditions derived in equations (26) - (29):

\[
\epsilon^* = \frac{\lambda_{min}}{2\eta\kappa\|P\|\Psi(T - h(u)) + \|B\|\Psi(T - h(u))}.
\]

Note that \(L(u, T)\) is minimized uniquely at \((T^*, u^*)\). It follows therefore that \((T^*, u^*)\) is a globally asymptotically stable equilibrium point for all \(\epsilon < \epsilon^*\), or all \(\Delta > \Delta^*\).
4. SIMULATION

In this section we present simulation results of our proposed control algorithm. For the simulations we consider a four-room building from an example in [Moore et al. (2011)], which is illustrated in Figure 1 below. Heat transfer to the ambient for all rooms is added to the model. In the figure, each double headed arrow represents a thermal connection between the two corresponding sides. The connection between two rooms through an open door is represented by a single resistance, and the same through the wall is represented using 3R2C wall model. The simulation results presented in this work have been obtained with two users (occupants) in room 4, and one user each in the other three rooms.

For this example model of four rooms and eight walls, we get 20 capacitive elements and 27 resistive elements. This gives us the dimensions of the incidence matrix, \( D \), for the model as \( 20 \times 27 \). Using the dimensions of the model in Figure 1, volumetric heat capacity values and thermal resistance values as per [Mukherjee et al. (2012)], we can obtain values for the matrices in equation (2). Using this information we simulate the model with ambient temperature at this information we simulate the model with ambient temperature at this information we simulate the model with ambient temperature at this information we simulate the model with ambient temperature at.

This is depicted in Figure 2 below. Also, in our simulation we assume a convex user discomfort function of the form:

\[
G_s(y_j) = \begin{cases} 
2(y_j - y_s^U)^2 & \text{if } y_j > y_s^U, \\
0 & \text{if } y_s^L \leq y_j \leq y_s^U, \\
2(y_j - y_s^L)^2 & \text{if } y_j < y_s^L,
\end{cases}
\]

where \( y_s^U \) and \( y_s^L \) are the upper and lower limit temperatures, respectively, of the user \( s \) located in zone \( j \).

![Fig. 1. Four room example model used for simulation. Each room has occupancy as illustrated in the figure.](image)

The results presented in Figures 3 and 4 are for a simulation run over a 5 hour period, when all the users get in at 7am and stay for a 5 hour period before breaking off for lunch hour. The results show that the temperature and control (heat) input converges. The converged temperatures were verified to minimize the weighted sum of the energy usage and aggregate user discomfort, subject to allowed lower and upper constraints on the temperature.

![Fig. 2. Linear probability distribution applied to user feedback at the lower and upper temperature limits.](image)

<table>
<thead>
<tr>
<th>Range (all users)</th>
<th>Low Temp. (°C)</th>
<th>High Temp. (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range (U1)</td>
<td>17</td>
<td>27</td>
</tr>
<tr>
<td>Range (U2)</td>
<td>19</td>
<td>22</td>
</tr>
<tr>
<td>Range (U3)</td>
<td>20</td>
<td>22</td>
</tr>
<tr>
<td>Range (U4)</td>
<td>21</td>
<td>24</td>
</tr>
<tr>
<td>Range (U5)</td>
<td>20</td>
<td>23</td>
</tr>
</tbody>
</table>

The evolution and convergence of the simulated system is dependent on the values of three major model parameters,
Fig. 3. Temperature evolution of the 4 room model over a 5 hour period with the rooms occupied throughout the period as per Figure 1. The user temperature preference is as per Table 2.

feedback gain (step size) parameter ($\eta$), user feedback weight parameter ($\gamma$), and heat cost parameter ($\Gamma$). The feedback gain (step size) parameter controls the size of change in $u$ in each step. Small values of $\eta$ would lead to a very slow convergence of the system to the desired temperature. With a high value of $\eta$ we can hit the desired range of temperature much faster, but at the cost of high temperature overshoots resulting in much higher total heat input. Also, a sufficiently high value of $\eta$ could violate the time scale separation assumption that is needed for stability. In our study, $\eta$ was tuned to obtain a reasonable trade-off. Another important parameter of the model, user weight parameter ($\gamma$), signifies the weight given to the penalty associated with the building occupant discomfort. A higher $\gamma$ would result in a high value of control input for a given user comfort feedback vector. This in turn would cause the temperature to change sharply as a result of user discomfort, but may also result in large overshoots as well as higher energy costs. In our simulations, $\gamma$ was tuned to obtain a reasonable trade-off between energy cost and user discomfort level. In this study we use a time varying heat cost factor $\Gamma$. Once the last user leaves the building we increase the value of $\Gamma$, so that heat input (energy) is minimized during non-occupant hours.

4.1 Multi-user, common temperature preference

The scenarios simulated in Figures 3 and 4, where the users all arrive at the same time, and stay in their respective rooms for several hours, do not represent a typical real world scenario. We next present simulation results on the model over a 48 hour period in a typical office environment. The building occupants move in and out as per the schedule shown in Table 3. Room 4 is occupied by two users $U4$ and $U5$. We consider two different cases for the users of room 4. First case corresponds to both the users of room 4 having a common range of comfortable temperature range as shown in Table 2. The temperature and control input over the 48 hour period is shown in Figures 5 and 6. In Figure 7 we show the corresponding user feedback input per room, obtained as the derivative of their discomfort function (level of discomfort that they express), as in (10). The simulation starts at 7am when the first user occupying zone 1 gets into office. The zone is at its initial temperature of 15°C. The user provides the feedback that it is cold which results in additional heat input to the building. One hour later (at 8 am) user occupying zone 2 gets in, and all other users come in at 9 am. The temperature quickly converges to the desired comfortable range and stays there until the users step out for lunch hour at 12 pm. At this point the heat input is reduced in the building and the temperature starts falling towards the ambient to conserve energy. Once the users get back in from lunch (at 1 pm) and provide feedback the heat is adjusted to quickly drive the temperature to comfortable range again. The heat input starts dropping again, as users start leaving starting 4 pm. At the end of day when the last user steps out (at 6 pm), heat input to the building is almost immediately reduced, and the temperature keeps falling overnight. Next day, users arrive in different order: zone 3 and 4 users at 8am, and zone 1 and 2 users at 9am. All of them step out for lunch hour at 12pm. Finally the users stop out at the end of day: zone 3 and 4 users at 5pm and zone 1 and 2 users at 6pm. The temperature follows the user occupancy pattern to optimize energy usage.

Figure 7 shows the user feedback history over the 48 hour period of model simulation. A positive (negative) value of feedback corresponds to the user feeling warm (cold) in the current temperature. Positive (negative) user feedback would result in cooling (heating) of the room in the next
4.2 Multi-user conflicting temperature preference

The second case corresponds to the users 4 and 5 (occupying room 4), having a conflicting temperature preference. The consolidated temperature range of all the users is shown in Table 4. The temperature evolution with the corresponding heat input in Figures 8 and 9. In this scenario since there is no common comfortable range for both the users of room 4, its not possible to satisfy both the users simultaneously. Hence, the temperature of room 4 settles between 23°C and 24°C, which minimizes the total discomfort level for the two users given their conflicting preferences. The overall heat consumed is higher when compared to the earlier case.

<table>
<thead>
<tr>
<th>Range (all users)</th>
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<tbody>
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<td>20</td>
<td>22</td>
</tr>
<tr>
<td>Range (U4)</td>
<td>21</td>
<td>23</td>
</tr>
<tr>
<td>Range (U5)</td>
<td>24</td>
<td>25</td>
</tr>
</tbody>
</table>

Table 4. Each user’s range of comfortable temperature. Users 4 and 5 occupying room 4 now have conflicting temperature preferences.

5. CONCLUSION AND FUTURE WORK

In this paper, we demonstrate that the building temperature and energy usage can be controlled successfully and efficiently through dynamic feedback from the users (occupants) based on their comfort levels. Under the reasonable assumption that user feedback is provided at a slower time scale as compared to the building temperature dynamics, our analysis shows that the proposed control algorithm results in the desired (optimal) trade-off between energy usage and user discomfort. The simulation study presented further shows that with effective tuning of a few parameters, the control algorithm attains a fast temperature convergence rate with reasonable energy expenditure. The binary form of the feedback that we have experimented


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