Control of piecewise linear hybrid systems

A piecewise linear hybrid system consists of two parts:

- An automaton \((Q, E_{cd} \cup E_{in}, f)\), where
  \(Q\) is the set of states, \(E_{cd}\) are events that are triggered by the continuous dynamics (guard), \(E_{in}\) are input events (externally triggered), \(f\) represents the transition function.

- For each \(q \in Q\) we define an affine system
  \[
  \dot{x}_q = A_q x_q(t) + B_q u(t) + a_q
  \]
  \[
  y(t) = C_q x_q(t) + D_q u(t) + c_q
  \]
\[ q^+ = f(q^-, x_k^-) \]

\[ x_k^+ = A_r(q^-, x_k^-, x_k^+ + b)(q^- + x_k^-) \]

When an event \( e \in E \) in or when the continuous state reaches the guard \( G_q(e) \), a transition occurs and the state is updated according to:

\[ x_k^+ = g_k(x_k^-) \]

with \( x_k \in X_k \) and \( v \in U \).
Assumptions:

- At any fixed time only a finite number of discrete transitions can occur (no livelock).
- On any finite interval only a finite number of discrete transitions can occur (non-zero)
exercise is said to be reachable.

If every state is reachable from the initial state, then the

such that by applying the input strings $e_i$ and input signals

\[
\forall i \leq \left\lfloor \frac{t}{e} \right\rfloor + 1 \rightarrow \cup, \; i = 1, 2, \ldots, n
\]

\[
(t_i, e_i), \; i = 1, \ldots, m, \; \text{if \; even, \; } t_i < t_j, \; j > i
\]

\[
(x_0, x_{x_0}, 0) \in A \times \text{If there exist two sequences}
\]

A state $(y_0, y_{x_0}) \in A \times x \in x$ is reachable from the initial state.

Reachability:
\[ \forall (r, g, b, c) \] 

\[ \exists x, y, z \in \mathbb{R}^+ \text{ s.t. } x^2 + y^2 + z^2 = 1 \]

A group (G, e, *)
A e e e a, g e a
\[
\{ (x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1 \} = \mathbb{D}
\]

An arbitrary set (or empty set) is defined to be a set of the form

\[
\mathbb{A} \subseteq \mathbb{R}^2
\]
For any \( x \in S \subseteq X \),

\[
\text{Cond}(S, X^2) = \bigcup_{x \in S} \text{Cond}(x, X^2)
\]

The set of all states \( x \in X \) from which \( x \) can be reached without leaving \( X \).

The controllability set (CCond) of \( (g, x) \in A \times X \) is...
Assume that there exists a collection of different sets of the form $A = \{ A(x, n) : x \in \mathbb{Z}, n \in \mathbb{N}, 2 \leq n \}$.
\[ A(\varrho, e, e^+ \leq \chi^+) \supset \bigcup_{\kappa \in \text{Eve}_{\vartheta, \chi}} A(\varrho, \kappa) \]

Assume that for every univ. set \( A(\varrho, e, e^+ \leq \chi^+) \),
The automaton is possibly non-deterministic.

\[
\text{for } (A(g, r, x, f, a), c) \in \text{context}(C_{g, r, x, f, a}) \quad \text{and}
\]

\[
\text{or } (A(g, r, x, f, a), c) \in \text{context}(C_{g, r, x, f, a})
\]

When the transition function \( f \) is defined as:

We can build an automaton \((A, E, \nu, \epsilon, f, a, 0)\),
Similar to the bisimulation algorithm, the construction of the finite set \( A \) can be done with an induction.

The second condition that can reach any set from \( A \) for any \((q, x) \in A \times x\), there exists an \( (q', k) \) such that

\[
A(q', k) = \text{const}(q, x) \]

For any \((q', x) \in A \times x\), there exists an \( A(q', x) \) such that

The situation \((A, E, \text{used}, f, a, A_0)\) is reachable, only if and only if:

Reachability and: The previously discussed high level systems reach.
\[ n \cdot x(t) \geq 0 \quad (\text{the curve is going out}) \]

\[ x(t) \in F \quad \iff \quad t = T \]

\[ x(t) \in \& \quad A \& \text{c}(0, T) \]

\[ t \in T, \forall n \in [0, T] \cup \text{such that: } \]

A polygonal line through the vertex \( T \)?

How to choose the control input such that the state exists?

Problem:
at other facts. Note that vector \( u_1 \) is parallel to the exit facet, and similarly

vertices of the polyhedral. This must be USED values such that the
necessary condition: Full dimension polyhedral.atham et al.
If \( P \) is a simplex, any point in \( D_n \) can be written as a

\[ \sum_{i=1}^{n} \lambda_i v_i = v, \quad \lambda_i \in [0,1], \sum_{i=1}^{n} \lambda_i = 1 \]

The convex hull is then defined as:

\[ \text{conv} \{ v_1, v_2, \ldots, v_n \} = \{ \sum_{i=1}^{n} \lambda_i v_i : \lambda_i \in [0,1], \sum_{i=1}^{n} \lambda_i = 1 \} \]
To apply the method for the simplex,

\[ v_2 \]

\[ v_1 \]

\[ v_0 \]

\[ v_0 \]

E.g.: in \( \mathbb{R}^2 \), triangulation.

For abstract polytopes (not necessarily simplicial), the

\[ \text{simplicial} \]

approach can be used by breaking down the polytope into simplices.
Optimal control of linear discrete-time hybrid systems
Consider the optimization problem based on the following cost

\[ J(u, x) = \sum_{k=0}^{\infty} L(x(k), u(k)) \]

for the system

for all \( x \in \mathbb{R} \), \( x(0) = x_0 \).
A good approximation.

Hence, taking large enough, we can get

Informal reasoning: if the infinite sum converges, then the two

\[ x(0) = x \]

\[ x = 0 \]

\[ \int_{n}^{n+1} = \lim_{k \to \infty} \int_{n}^{n+1} x_k(u) \, du \]

An approximation is by any finite horizon.

Because of the infinite time horizon, the solution is difficult.
Algorithm:

The idea of finite horizon is combined with accuracy horizon.

The problem is more tractable.

The optimisation must be performed, i.e., \( \text{u}(t), \text{u}(t-1), \ldots, \text{u}(0) \) on which the horizon truncated.
Apply \( u(c_0) \) to \( c \), and so on.

Compute the objective \( u(c_1), \ldots, u(c_T) \).
Finite time constrained optimal control:

Constraints: \[ U_{\text{min}} \leq U(t) \leq U_{\text{max}} \]
\[ x_{\text{min}} \leq x(t) \leq x_{\text{max}} \]

defines a polyhedron \( D \)

\[ x(t+1) = A_i x(t) + b_i u(t) + f_i \quad \text{if} \]

\[
\begin{bmatrix}
  x(t) \\
  u(t)
\end{bmatrix} \in \tilde{X}_i, \quad \tilde{X}_i := X_i \cap D
\]
If \( p = 2 \), \( \| \Theta \|_2 \sim (\Theta^T \Theta)^{-\frac{1}{2}} \) quadratic cost.

\( x \) is the desired input function.

\( x^e \) is the desired equilibrium state.

For example: \( p \) is unmixing, problem specific norm denotation.

\[
\begin{align*}
\| p(x(t)) - x^e \|_p^p \\
\| \Theta (x(v)) - x^e \|_p^p \\
\| R(u(x)-u^e) \|_p^p
\end{align*}
\]

\[
\int_{u_{-1},x(0)}^{u_0,f(x)} \frac{1}{t-1} dt
\]

Define the cost function.
\( u(t) = F(\xi(t), x(t) + g(t)) \forall x(t) \in P(t) \)

The solution of the optimal control problem, given the form

\[
\min \quad P(0) + \int_{0}^{\infty} \left( x(t)^{T} - x_{e}^{T} \right)^{2} dt + \sum_{t=0}^{\infty} \mathbb{E} \left( (x(t+1) - x_{e}^{T})^{2} + (u(t) - u_{e}^{T})^{2} \right)
\]

with a, p, 0, K > 0.
where $P(l, t) = \text{a partition of the state space},$ given by

$$\{(l_1, t_1) \leq \ldots \leq (l_i, t_i) \leq \ldots \leq (l_N, t_N)\}$$

- Transform the dynamics of the system into mixed logic

The solution is obtained:

$$\text{dynamics (MLC0)}$$
Tool: MPT toolbox for MATLAB

A Mixed Integer Quadratic Programming.
Together with the quadratic cost function, the problem becomes