When a transition occurs, the continuous state hits the boundary of an invariant. A transition can be caused by a transition. A PDMP is a hybrid stochastic process. The continuous dynamics is non-stochastic. Stochastically, when the system performs a transition.
Let $\mathcal{D}$ be the union of the null space of $\mathfrak{d}$.

We now define the weighted space as

\[ D = \bigcup \{ \mathfrak{d} \times \mathfrak{c} \} \]

\[ \mathfrak{c} \subset \mathbb{R}^d \text{ is the invariant of the location,} \]

each location.

\[ \mathfrak{d} : \mathbb{E} \to \mathrm{Nil} \text{ is the dimension of the continuous state space} \]

\[ \mathfrak{a} : \mathbb{H} \times \text{set of directed edges} / \text{locations} \]

\[ A \text{ is the set of directed edges / locations} \]

\[ \text{Formally:} \]

\[ H = (\mathfrak{a}, \mathfrak{d}, \mathfrak{x}, f, \mathfrak{t}, \mathfrak{n}, \mathfrak{e}) \]
\[ p \text{ (A sheet is cut to \( R \) from \( R', d \))} = R(R', d) \]

\( R \) is a probability measure in \( D \), such that for any \( R \in D \),
\( R \) is the transition probability. For any \( A \), such that \( A \in D \),
\[ \lambda : 0 \rightarrow A + \] is the rate of the Poisson process.

\[ X = f(x) \]

Init is the distribution of \( x \) in its initial state.

\[ \text{Dynamics of Location} \]
\[
\phi(x) = \frac{\partial}{\partial \phi} \phi \quad \text{for} \quad x \in \mathbb{R}.
\]

Thus, \( \phi \in \mathbb{R} \).

Define \( \phi(x) \) to be the solution of \( x = f(x) \), with initial condition \( \phi \).

Assume that the function is \( \phi \).

Define \( x = f(x) \), and the value \( x \) is a function of \( x \).

Poisson process with random variables.
If \( X \) is exponential: \( F(x, t) = e^{-xt} \), i.e. \( X \sim \text{exp} \).

The Poisson process has not been noticed after time \( t \).

Interpretation: \( F(x, t) \) is the probability that the state is 2, starting the state at 1.

Definition: \( F(x, t) = \exp \left( -\int_{t}^{\infty} \lambda(\phi) \, d\phi \right) \)
To simulate: generate a random variable $T$ from the distribution $\phi_t(x)$.

Given by $f(x, t)$, if $T < t$ then $X_t$ jumps at time $T$.

Generally, if the invariant is not unbounded:

Define $t^*(x) = \inf \{ t > 0 | \phi_t(x) \neq x \}$.
\[
E \left( \{ A(t) \mid t = d \} = \{ c(d) \} + E \int_{t=0}^{t} A \left( \{ A(t) \mid t = d \} = \{ c(d) \} \right) \, dt \right)
\]

Pytchnik's formula:

\[
\lim_{t \to 0} \frac{E(\{ A(t) \mid t = d \} = \{ c(d) \}) - E(\{ c(d) \})}{t}
\]

where \( b \) is given.

For any \( \delta > 0 \), the generator of the stochastic process is the infinitesimal generator.
\( R(\text{dx},d) = \int P(dx) \, dx \)

Density

\[ \int \left( f(x) - \{f(x) \} \right) \, dx \]

Infinite measure

\[ A \{ \frac{xf}{e} \} = \frac{1}{e} \int \nu(x) \, dx \]

The infinitesimal generator of a process is given by:
This is no unusual, so there is no jump associated with bounds.

Two discrete states are featured by process progress.

Evolve within a stochastic differential equation, and transitions be.

A general is a hybrid stochastic process, where the continuous state

Switching Diffusion Process (SDP)
Formally, an SDP is $H = (Q, X, f, S_{\text{init}}, \sigma, \gamma)$ where:

- $Q$: The set of discrete states/locations
- $X = \mathbb{R}^n$, the continuous state space
- $f: \mathbb{R}^n \to \mathbb{R}^n$, a vector field that characterizes the anti-probabilistic dynamics

The initial state is $\gamma$, the transition rate for process $j$ from location $i$ to location $j$. $\gamma_{ij} = -\sum_{j \neq i} \gamma_{ij}$.
Continuous states are not real.

Y2 = Y3

Y1 = Y2

Loc 1

Loc 2

Loc 3
The influence of an asymmetric Brownian motion.

We introduce a vector of an inhomogeneous Brownian motion.

\[ dx_t = \left( g'(x_t) dt + \sigma (x_t) dW_t \right) \]

The stochastic differential equation is given by:
a probability distribution.

When a transition occurs, the continuous state is not affected.

Any lift is boundary of the invariant

transition between actions, happens when the continuous feature
decays a stochastic differential equation.

In a stochastic hybrid system (SHS), the continuous state

Stochastic Hybrid System (SHS)
\[ \bigcup \left( G(x, y) = \text{dom}(G) \right) \]

\[ C(x, y) \cup (G(x, y) = \text{dom}(G)) = P, \text{ all } x \geq y \]

**Boundary**

\[ G(x, y) \subseteq \text{dom}(G) \rightarrow \text{boundary} \]

\[ G : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R} \text{ is an } \text{ smooth } \text{ set} \]

Int is the distribution of the 'ink' edge

\[ f, g : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R} \text{ are } \text{ vector fields} \]

\[ \text{Dom}(G) \text{ is the invariant of } \text{ location} \]

\[ x = \mathbb{R} \text{ is the continuous state space} \]

\[ \mathbb{R} \text{ is the set of locations} \]

\[ H(x, y) \text{ dom, } f, g, \text{Init, } G, \mathbb{R}, \text{ where} \]

**Formally, an sis is:***
\[ \lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} f(x_i) \Delta x \]

The continuous dynamical system is given by:

\[ R(g, x, \tau) \]

The probability is given by

\[ R(g, 0, x) \]

Domain (g, 0)

Domain (g, 0)

The probability measure in \( R((g, 0, x), x \in G(g, 0)) \) is the distribution on

\( R \) is the left distribution.
\[ R \text{ is the probability measure corresponding to the vector.} \]

\[ \int \mathbb{B}(p) - \int \mathbb{Q}(p) \mathbb{R}(dp, r) \]

\[ + \left( \frac{x e^{-x}}{x} e^{-x} \right) (\mathbb{g}(x) - \mathbb{g}(p)) + \frac{2}{\sqrt{\pi}} \mathbb{g}(p) \mathbb{f} \mathbb{e} \frac{x e^{-x}}{x} \right) = \mathbb{g}(p) \]
Summary:

<table>
<thead>
<tr>
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<th>Diffusion</th>
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<tr>
<td>PDMP</td>
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<td>SHS</td>
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<td>❌</td>
<td>✔️</td>
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</tbody>
</table>
In PHS, the vector fields, mass rates, and transition rates are performed functions.

A PHS is composed by an SDP.

Poisson processes. SDE, which transitions between Euclidean and functional space, which in evolution of the continuous phase follows an exponential distribution. This is a Poisson process, which describes the hybrid system. However, due to

Pohang Research Standard Hybrid System (PHS)
If $f$ is a polynomial, $A_f$ is a polynomial.

Assume that $f, d, X$ are polynomials.

$$\forall e \in \mathbb{R}$$

$$\forall f : \mathbb{R}^* \to \mathbb{R}^*$$

$$A_f = \int_{\mathbb{R}} \frac{e^x}{f(x)} (\frac{\partial}{\partial x} X) + \int_{\mathbb{R}} \frac{\partial}{\partial x} f(x) = \frac{e^x}{f(x)}$$

$$A_f = (\frac{\partial}{\partial x} X)$$

The infinitesimal generator of an SOP is given by:

Recall the infinitesimal generator of an SOP:
The idea: the higher order moments are approximated by a polynomial (monomial) function of the lower order ones. This is possible by truncating the higher order moments.

For finite dimensional $n$-th order moments, $E[f(x)^n]$ is a linear function. Thus, we define $f$ as the finite vector of monomials, the term $E[f(x)^n]$ as the polynomial:

$$E[f(x)^n] = \frac{d^n}{df} E[\phi(f)] = E[\phi(f)]$$