ECSE 6420: Nonlinear Control Systems

Homework set 2. Due date: 27 Feb 2009

Points: Problem 1 = 2+2+2+2+5 pts, Problem 2 = 20 pts, Problem 3 = 6+8+10+10 pts, Problem 4 = 5+10 pts, Problem 5 = 5+13 pts

Problem 1. Determine which of the following functions are locally Lipschitz and globally Lipschitz:
(a) $f(x) = x^2$,
(b) $f(x) = |x|$,
(c) $f(x) = x + 4\cos x$,
(d) $f(x) = \tan(x)$,
(e) $f\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -x_1 + |x_2| \\ x_2 - x_1 \end{pmatrix}$

Problem 2. Consider a model of two mobile robots, whose position on a 2D plane is given by $x_1(t)$ and $x_2(t)$ respectively. Thus $x_1(t), x_2(t) \in \mathbb{R}^2$. A kinematic model of the robots is given as follows:

$$\dot{x}_1 = f(x_2 - x_1) + u_1, \tag{1}$$
$$\dot{x}_2 = f(x_1 - x_2) + u_2, \tag{2}$$

where $f : \mathbb{R}^2 \to \mathbb{R}^2$. The functions u_1 and u_2 are external inputs, which are bounded signals. Suppose that f is continuously differentiable and there exists a bound $K > 0$ such that

$$\sigma_{\max}\left(\frac{\partial f}{\partial x}\right) < K,$$

for all $x \in \mathbb{R}^2$, where $\sigma_{\max}$ denotes the largest singular value. Using contraction mapping theorem, show the local and global existence and uniqueness of the solution of (1-2).

Problem 3. A model of the tunnel-diode circuit that was discussed earlier in the course can be written as:

$$\dot{x} = E - 10x - y,$$
$$\dot{y} = x - 2y^3 + 9y^2 - 12y,$$

where $E > 0$ can be considered as a system parameter, taking its value in the interval $[50.5, 51.5]$.
(a) Plot the nullclines $\dot{x} = 0$ and $\dot{y} = 0$ in the box $(0 \le x \le 10, 0 \le y \le 3)$ for several values of E in the given interval.
(b) Take the origin as initial condition, simulate the trajectories of the system

in the time interval $t \in [0, 10]$, for several values of E in the given interval.
(c) Compute the sensitivity of trajectories in (b) with respect to E.
(d) Discuss the different sensitivity results, and relate them to the plot in (a).
Note: For points (a)-(c), make sure that you pick at least 5 different values of E spread relatively uniformly in the interval $[50.5, 51.5]$.

Problem 4.(a) Prove that for any $x, y \in \mathbb{R}$, the following holds

$$x^2y^3 \leq x^4 + y^6.$$

(b) Using the result of part (a), prove that the origin in the following system is globally asymptotically stable

$$\begin{aligned}\dot{x} &= -x^3 + xy^3,\\ \dot{y} &= -y.\end{aligned}$$

Problem 5. Consider the system

$$\begin{aligned}\dot{x}_1 &= -x_1 + x_1x_2,\\ \dot{x}_2 &= -\gamma x_1^2, \ \gamma > 0.\end{aligned}$$

(a) What is the set of equilibria of the system?
(b) Using the (LaSalle's) invariance principle, determine the stability of this set.