ECSE 6420: Nonlinear Control Systems

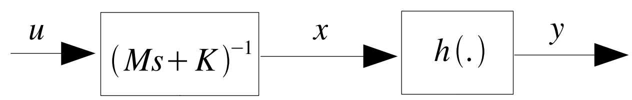
Homework set 4. Due date: 9 April 2009

Points: Problem 1 = 25 pts, Problem 2 = 25 pts, Problem 3 = 10+15 pts, Problem 4 = 25 pts

Problem 1. (Problem 6.5) Consider the system represented by the following block diagram, where $u, y \in \mathbb{R}^p$, M and K are positive definite symmetric matrices, $h \in [0, K]$ and

$$V(x) := \int_0^x h^T(\sigma) M \, d\sigma \geq 0, \forall x.$$

Show that the system is output strictly passive. Hint: Use $V(x)$ as storage function.



Problem 2. (Problem 6.7) Show that the transfer function

$$H(s) = \frac{s+b}{s^2 + a_1 s + a_2}$$

is strictly positive real if and only if all coefficients are positive and $b < a_1$.

Problem 3. (Problem 7.3) Consider the system

$$\dot{x}_1 = -x_1 - h(x_1 + x_2),$$
$$\dot{x}_2 = x_1 - x_2 - 2h(x_1 + x_2),$$

where h is a smooth function satisfying

$$yh(y) \geq 0, \forall y,$$
$$h(y) = \begin{cases} c, & y \geq a_2 \\ 0, & |y| \leq a_1 \\ -c, & y \leq -a_2 \end{cases},$$
$$|h(y)| \leq c, \text{ for } a_1 < y < a_2 \text{ and } -a_2 < y < -a_1.$$

(a) Show that the origin is the unique equilibrium point.

(b) Using the circle criterion, show that the origin is globally asymptotically stable.

Problem 4. (Problem 9.24) Study the stability of the origin of the system

$$\dot{x}_1 = -x_1^3 - 1.5x_1|x_2|^3$$
$$\dot{x}_2 = -x_2^5 + x_1^2 x_2^2$$

by using composite Lyapunov analysis.