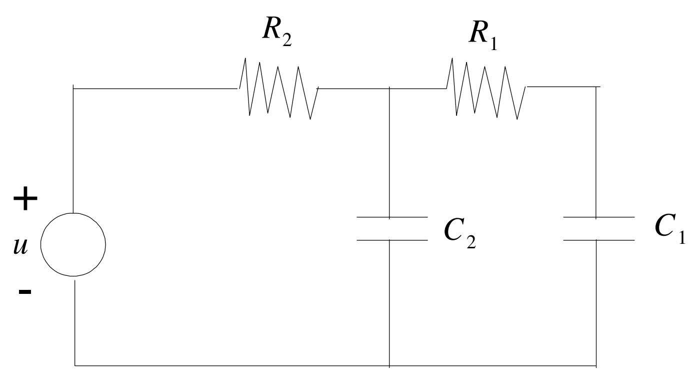
ECSE 6420: Nonlinear Control Systems

Homework set 5. Due date: 5 May 2009

Points: Problem 1 = 10+10 pts, Problem 2 = 10+10+10 pts, Problem 3 = 10+10 pts, Problem 4 = 8+15+7 pts

Problem 1. (Problem 11.1 and 11.2) Consider the RC circuit below.



a) Suppose that the capacitor C_2 is much smaller than C_1, while $R_1 = R_2 = R$. Represent the system in the standard singularly perturbed system. Analyze and define the fast and slow dynamics.
b) Suppose that the resistor R_1 is much smaller than R_2, while $C_1 = C_2 = C$. Represent the system in the standard singularly perturbed system. Analyze and define the fast and slow dynamics.

Problem 2. (Problem 12.2) Design a state feedback controller and an output feedback controller to stabilize the origin for each of the following systems:
a)

$$\begin{aligned} \dot{x}_1 &= x_1 + x_2, \\ \dot{x}_2 &= 3x_1^2 x_2 + x_1 + u, \\ y &= -x_1^3 + x_2. \end{aligned}$$

b)

$$\begin{aligned} \dot{x}_1 &= x_1 + x_2, \\ \dot{x}_2 &= x_1 x_2^2 - x_1 + x_3, \\ \dot{x}_3 &= u \\ y &= -x_1^3 + x_2. \end{aligned}$$

c)

$$\begin{aligned}
\dot{x}_1 &= -x_1 + x_2, \\
\dot{x}_2 &= x_1 - x_2 - x_1 x_3 + u, \\
\dot{x}_3 &= x_1 + x_1 x_2 - 2x_3 \\
y &= x_1.
\end{aligned}$$

Problem 3. Consider the system

$$\dot{x}_1 = -x_1 + x_2 - x_3, \tag{1a}$$

$$\dot{x}_2 = -x_1 x_3 - x_2 + u, \tag{1b}$$

$$\dot{x}_3 = -x_1 + u, \tag{1c}$$

$$y = x_3 \tag{1d}$$

a) What is the relative degree (r) for this system? Is it input-output linearizable? If so, find $\alpha(x)$ and $\gamma(x) \neq 0$ such that

$$\frac{d^r y}{dt^r} = y^{(r)} = \alpha(x) + \gamma(x)u. \tag{2}$$

b) Is the system (1) minimum phase? (Hint: First find the zero dynamics by setting $y \equiv 0$. Is the origin of the zero dynamics asymptotically stable?)

Problem 4. We consider the full state feedback linearization of the system in (1).

a) Suppose you know that (1) is full state feedback linearizable. Find an output $\tilde{y}$, such that the relative degree from u to $\tilde{y}$ is 3, the number of all the states. When is this relative degree well-defined?

b) Find a change of variables $z = T(x)$ that linearizes the state equations. Is this change of variables $z = T(x)$ a diffeomorphism? (Hint: By choosing z properly, you may want to write the transformed state equations as

$$\begin{aligned}
\dot{z}_1 &= z_2 \\
\dot{z}_2 &= z_3 \\
\dot{z}_3 &= \alpha(x) + \gamma(x)u.)
\end{aligned}$$

c) Based on b), how would you choose a simple control u to stabilize $z = 0$? Does the stabilization of $z = 0$ imply the stabilization of $x = 0$?