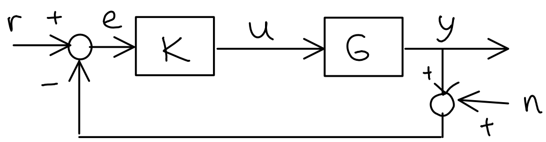
Some bits about Multivariable H_∞ synthesis (section 3.5.7)



Recall that the transfer matrix from:

r to e is $(I+GK)^{-1} = S$

r to u is $K(I+GK)^{-1} = KS$

n to y is $-(I+GK)^{-1}GK = -T$

Mixed synthesis $S/KS/T$

$$N = \begin{bmatrix} W_P S \\ W_u KS \\ W_T T \end{bmatrix}$$

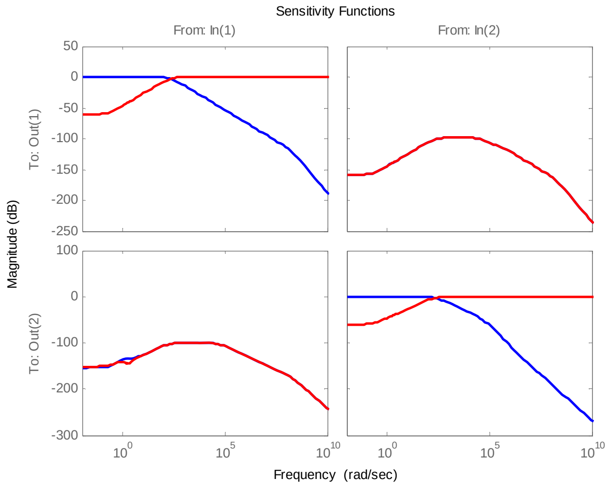
Goal: $\|N\|_\infty \le 1$

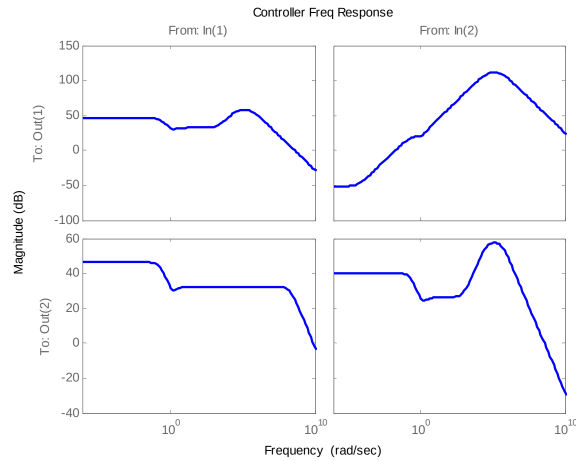
The weights can be taken as diagonal matrices, with diagonal entries formulated as in SISO case.

Example: Apply H_∞ synthesis for the MIMO problem discussed in the previous lecture, say for desired bandwidth = 100 rad/s

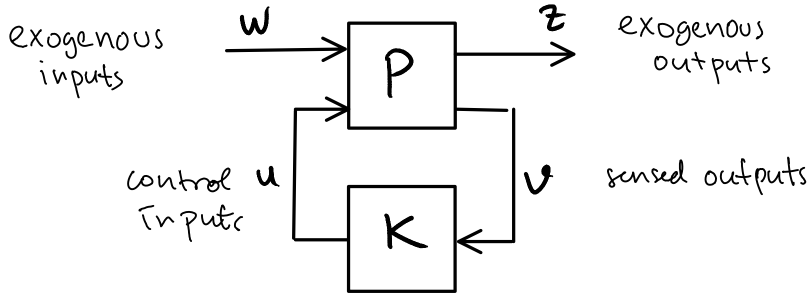
Formulation: $\|W_P S\|_\infty \le 1$, with $W_P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \dfrac{s/M + w_c}{s + w_c A}$

$w_c = 100$, $A = 10^{-3}$, $M = 1.05$





General Control Problem Formulation



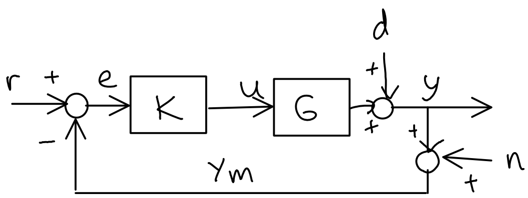
General idea:

w and z are used in characterizing performance

u and v are used in interconnection with controller

Control Problem: Design K such that the TF from w to z is small.

Examples: 1 DOF Feedback control



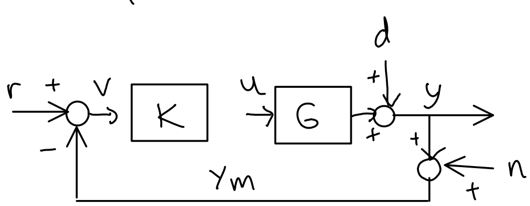
Identify the general variables:

$W = \begin{bmatrix} d \\ r \\ n \end{bmatrix}$; $z = y - r$

$u = u$; $v = e = r - y - n$

The generalized plant model consist of a TF from $\begin{bmatrix} w \\ u \end{bmatrix}$ to $\begin{bmatrix} z \\ v \end{bmatrix}$

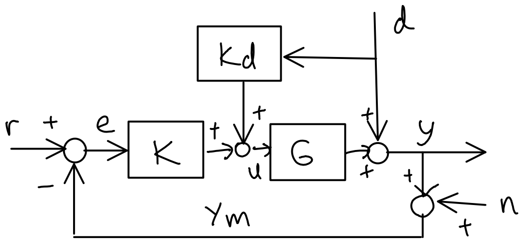
To compute this, cut the control loop.



$$z = y - r$$
$$ = Gu + d - r$$
$$v = r - Gu - d - n$$

Thus: $\begin{bmatrix} z \\ v \end{bmatrix} = \begin{bmatrix} I & -I & 0 & G \\ -I & I & -I & -G \end{bmatrix} \begin{bmatrix} d \\ r \\ n \\ u \end{bmatrix}$

Example: Disturbance feedforward control



Identify the general variables:

$$w = \begin{bmatrix} d \\ r \\ n \end{bmatrix} ; \quad z = y - r$$

$$u = u ; \quad v = \begin{bmatrix} r - y - n \\ d \end{bmatrix}$$

$$z = y - r = Gu + d - r$$

$$v = \begin{bmatrix} r - y - n \\ d \end{bmatrix} = \begin{bmatrix} r - Gu - d - n \\ d \end{bmatrix}$$

The transfer matrix:

$$\begin{bmatrix} z \\ v \end{bmatrix} = \begin{bmatrix} I & -I & 0 & G \\ -I & I & -I & -G \\ I & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix}$$

Partitioning P : We often partition P as follows:

$$z = P_{11} w + P_{12} u$$

$$v = P_{21} w + P_{22} u$$

Once we close the loop with $u = Kv$, the TF from w to z is

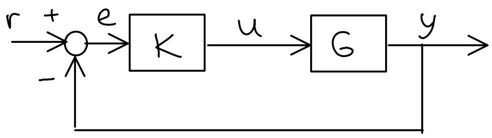
$$z = P_{11} w + P_{12} K v$$

$$v = P_{21} w + P_{22} K v \rightarrow v = (I - P_{22} K)^{-1} P_{21} w$$

$$z = \underbrace{\left[P_{11} + P_{12} K (I - P_{22} K)^{-1} P_{21} \right]}_{N = F_\ell(P, K)} w$$

$F_\ell(P,K)$ is called the lower Linear Fractional Transform (LFT) of P with K as the parameter. Controller synthesis amounts to finding K that will make N small

Example: S/T/KS mixed H_∞ synthesis



Find K that minimizes $\|N\|_\infty$,

where $N = \begin{bmatrix} W_P S \\ W_u KS \\ W_T T \end{bmatrix}$

Recall that: S is the TF from r to e

KS is the TF from r to u

T is the TF from r to y

Thus: use the general variables: $w = r$; $z = \begin{bmatrix} W_P \cdot e \\ W_u \cdot u \\ W_T \cdot y \end{bmatrix}$

$u = u$

$v = e$

The generalized Plant TF: $z = \begin{bmatrix} W_P(r - Gu) \\ W_u \cdot u \\ W_T Gu \end{bmatrix}$; $v = e = r - Gu$

$P_{11} = \begin{bmatrix} W_P \\ 0 \\ 0 \end{bmatrix}$; $P_{12} = \begin{bmatrix} -W_P G \\ W_u \\ W_T G \end{bmatrix}$; $P_{21} = I$; $P_{22} = -G$

Let's verify with LFT formula:

$$N = P_{11} + P_{12} K (I - P_{22} K)^{-1} P_{21}$$

$$= \begin{bmatrix} W_P \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -W_P G \\ W_u \\ W_T G \end{bmatrix} K (I + GK)^{-1} \cdot I$$

$$= \begin{bmatrix} W_P - W_P T \\ W_u KS \\ W_T T \end{bmatrix} = \begin{bmatrix} W_P S \\ W_u KS \\ W_T T \end{bmatrix}$$

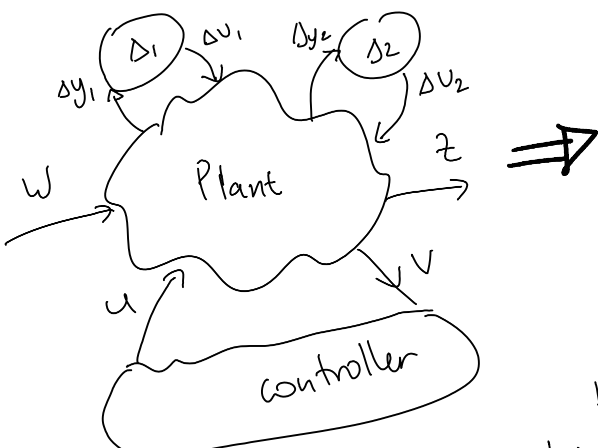
In MATLAB, given P, K can be synthesized using "hinfsyn"
Note: Not all stacked TF can be represented as LFT. Read Section 3.8.7

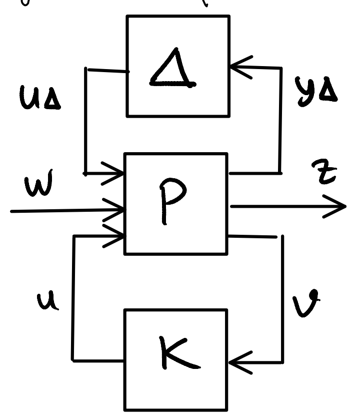
Including Model Uncertainty

General idea: The plant model might have uncertainties due to various factors such as parameter uncertainty, unmodeled dynamics, etc

The philosophy of robust controller synthesis is to design controllers for which the plant is stabilized regardless of uncertainty. This will be discussed later.

The way to include the uncertainty in the generalized plant model:





Here Δ is a diagonal transfer matrix capturing the individual Δ_i blocks

Δ_i are typically normalized such that $\|\Delta_i\|_\infty \leq 1$
Once we close the control loop, assume that we have a TF:

$$\begin{bmatrix} y_\Delta \\ z \end{bmatrix} = \begin{bmatrix} N_{11} u_\Delta + N_{12} w \\ N_{21} u_\Delta + N_{22} w \end{bmatrix}$$

Given Δ, the TF from z to w can be found through the Upper Fractional Transformation: $F_u(N, \Delta) = N_{22} + N_{21}\Delta(I - N_{11}\Delta)^{-1} N_{12}$