High gain feedback

If the loop TF $L = GK$ is high gain and the system is stable, then:

$$\left.\begin{aligned} S &= (I + GK)^{-1} \approx 0 \\ T &= I - S \approx I \end{aligned}\right\} \text{ perfect tracking, zero error}$$

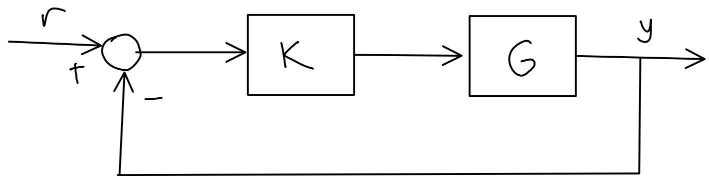
Later we will see that high gain feedback is not always feasible, because of stability issue.

Closed Loop Stability

Recall that stability is determined by the poles of the transfer function.
Observe that the poles of S and T are the same. Why?

Consider SISO systems: $S = \frac{1}{1+L}$. The poles of S are the zeros of $(1 + L(s))$

Example:



$G(s) = \frac{s-1}{s+1}$; $K(s) = \frac{k}{s+2}$

$$1 + L(s) = 1 + G(s)K(s) = 1 + \frac{s-1}{s+1}\cdot\frac{k}{s+2}$$

$$= \frac{(s+1)(s+2) + k(s-1)}{(s+1)(s+2)}$$

The zeros of $L(s)$: $(s+1)(s+2) + k(s-1) = 0$

$$s^2 + 3s + 2 + ks - k = 0$$

$$s^2 + \underbrace{(3+k)}_{b}s + \underbrace{(2-k)}_{c} = 0$$

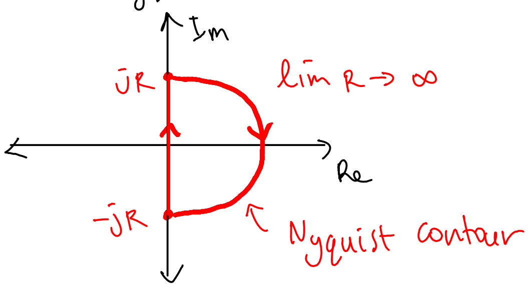
Routh-Hurwitz criteria for 2^{nd} degree polynomials:
stable iff $b>0$ and $c>0$, thus: $3+K>0$ and $2-K>0$

$\boxed{-3<K<2}$

$\boxed{\text{(re)Read about Routh-Hurwitz test!}}$

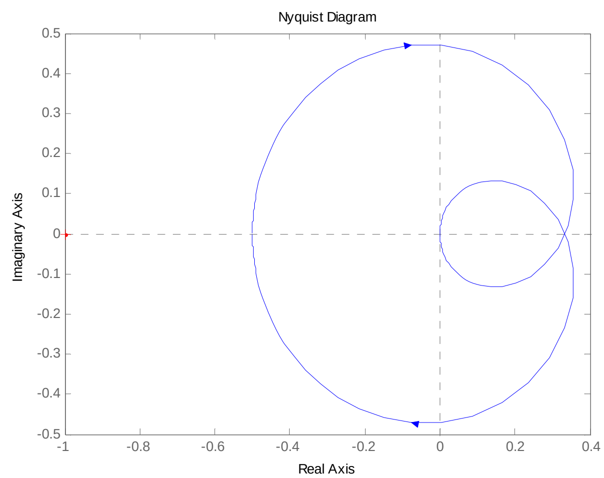
Another stability criterion/method: Nyquist criterion

Nyquist plot: Plot $L(s)$ on the complex plane as s follows the Nyquist contour



Use the same example as above with gain $K = 1$

Use MATLAB command "nyquist"
↓



Nyquist criterion:
clockwise encirclement of (-1) = # unstable CL poles − # unstable open loop poles

From our example:
0 = # unstable CL poles $- \; 0$, thus the CL system is stable. However, if the gain K is increased, -1 will be encircled, and thus the CL system is unstable.

Bode Criterion: If $\angle L(j\omega)$ crosses $-180°$ only once from above at $\hat{\omega}$ (thus $\angle L(j\hat{\omega}) = -180°$) then

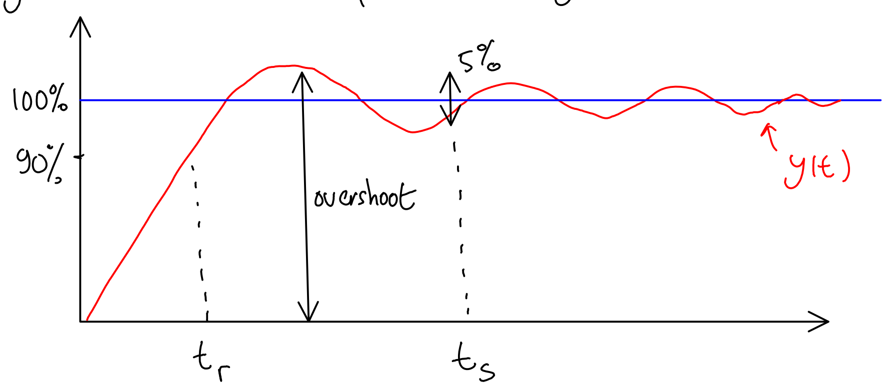
$$\text{stability} \Leftrightarrow |L(j\hat{\omega})| < 1$$

$$\text{Gain margin} = \frac{1}{|L(j\hat{\omega})|}$$

Interpretation: minimum additional loop gain to cause instability

Close Loop Performance Criteria

Typical step response of the CL system:



- t_r: rise time: time to reach 90% of final value
- overshoot: % of excess of final value
- t_s: settling time: time to settle into within ±5% of final value
- steady state error/offset: the gap between the reference and final value
- total variation (TV):

$$\int_0^\infty \left|\frac{dy}{dt}\right| dt$$

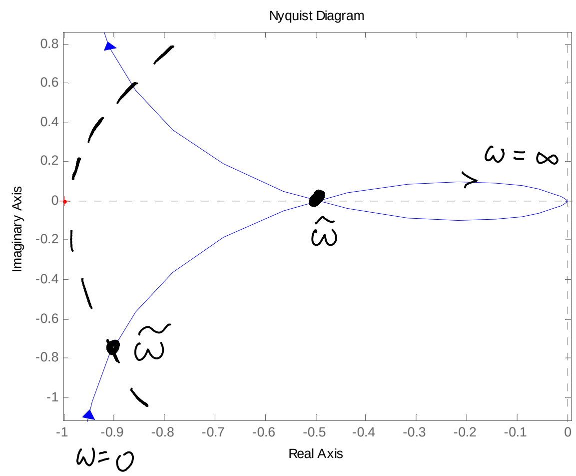
Frequency Domain Performance

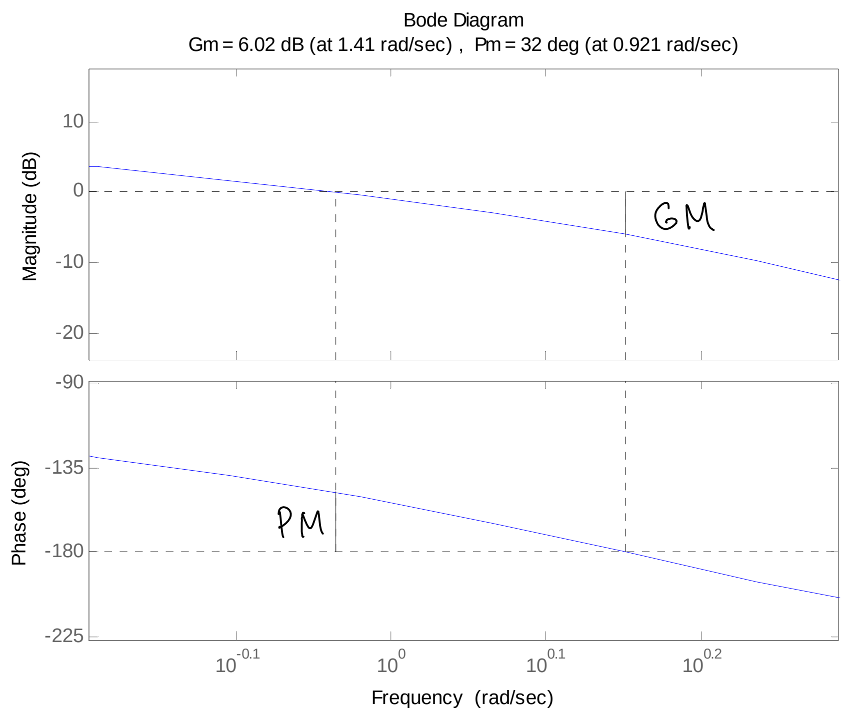
- Gain margin
- Phase margin

Example: $L(s) = \dfrac{2}{s(s^2+2s+2)}$

$$\boxed{GM: \frac{1}{|L(j\hat{\omega})|}}$$

$$\boxed{PM: \angle L(j\tilde{\omega}) + 180^\circ}$$





Peak Criteria: It is desirable to curb the peak (maximum gain) of S and T.

$$M_S = \max_{\omega} |S(j\omega)| \quad ; \quad M_T = \max_{\omega} |T(j\omega)|$$

Note that because $S + T = 1$, $|M_S - M_T| \leq 1$

Relationship between stability margin and peak criteria (p.36)

$$GM \geq \frac{M_S}{M_S + 1} \quad ; \quad PM \geq 2\arcsin\left(\frac{1}{2M_S}\right) \geq \frac{1}{M_S} \text{ (rad)}$$

$$GM \geq 1 + \frac{1}{M_T} \quad ; \quad PM \geq 2\arcsin\left(\frac{1}{2M_T}\right) \geq \frac{1}{M_T} \text{ (rad)}$$

- Read 2.4.4 (p.37)

Bandwidth

Closed loop bandwidth, ω_B is the frequency where $|S(j\omega)|$ first crosses $-3\,dB = 1/\sqrt{2}$ from below

Large bandwith → better capability to track "fast" reference signals