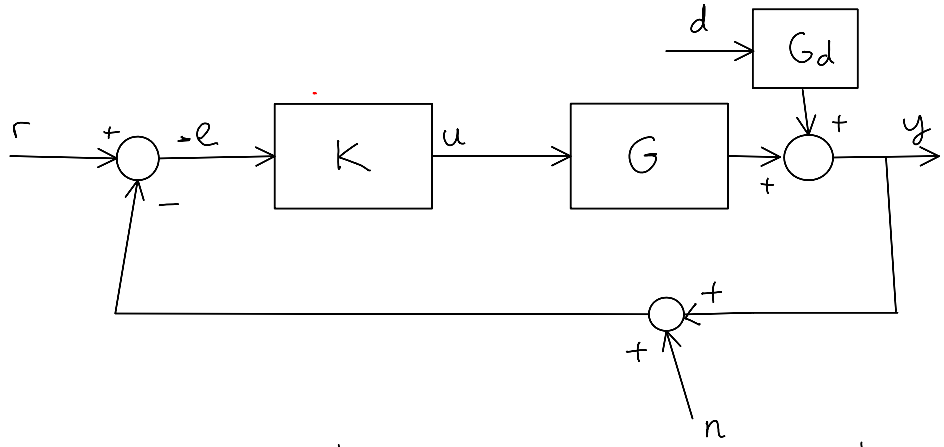
Loop Shaping : Shaping the loop transfer function $L = GK$



$$e = -(I+L)^{-1} r + (I+L)^{-1} G_d \cdot d - (I+L)^{-1} L n$$
$$= -Sr + SG_d \cdot d - Tn$$

Trade off : large $|L| \to$ small $|S| \to$ good tracking , but

$T = I - S$, so if $S \approx 0$, $T \approx I \to$ poor noise rejection

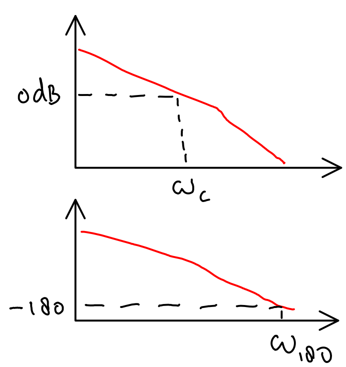
General idea :

- Large $|L|$ for low frequency range $\to$ better tracking for low freq reference
- Small $|L|$ for high frequency range $\to$ good high frequency noise rejection

Open loop bandwidth: ω_c is the frequency where $|L(j\omega)|$ first crosses 1 (or 0dB) from above

Recall Bode criterion: $|L(j\omega_{180})| = |L(j\hat{\omega})| < 1$

Thus; $\omega_c < \omega_{180}$



Other considerations

- Large $|K|$ means large control input $\rightarrow$ undesirable
- Steady state error : Suppose that $r(t) = 1(t) \leftarrow$ step function

$$R(s) = \frac{1}{s}$$

$$E(s) = \mathcal{L}\{e(t)\} = -\frac{1}{s} \cdot \frac{1}{1+L(s)} \rightarrow \text{ignore disturbance \& noise}$$

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{-1}{1+L(s)}$$

$\rightarrow$ for zero steady state error, $L(s)$ must have a pole at $s=0$

- sometime high gain feedback is necessary for stability

Example: $G(s) = \frac{1}{s-1}$; $K(s) = \frac{K(s+1)}{s}$

$$L(s) = G(s)K(s) = \frac{K(s+1)}{(s-1)\cdot s} \rightarrow 1 + L(s) = \frac{(s-1)s + K(s+1)}{(s-1)s}$$

Close loop poles : $1+L(s) = 0 \rightarrow s^2 - s + Ks + K = 0$

Stable iff $K > 1$ and $K > 0$ (Routh Hurwitz)

Example: Study example 2.8
Feedback amplifier in HW1

Inverse Model based Controller

A good loop transfer function for reference tracking looks like :

$$L(s) = \frac{\omega_c}{s} \rightarrow S(s) = \frac{1}{1+L(s)} = \frac{s}{s+\omega_c}$$

- Openloop and close loop bandwidth $= \omega_c$
- zero steady state tracking error for step function.

Also, $|S(j\omega)| = \left| \frac{j\omega}{j\omega + \omega_c} \right| = \frac{\omega}{\sqrt{\omega^2 + \omega_c^2}} = \frac{1}{\sqrt{1 + \frac{\omega_c}{\omega}}} \rightarrow M_S = 1$

$$|T(j\omega)| = \frac{\omega_c}{\sqrt{\omega^2 + \omega_c^2}} \rightarrow M_T = 1$$

If $r(t) = 1(t) \rightarrow E(s) = -S(s)\,R(s) = \frac{-1}{s + \omega_c}$

$$e(t) = -e^{-\omega_c t}, \quad t \geq 0$$

Thus, the speed of the response depends on the bandwidth.

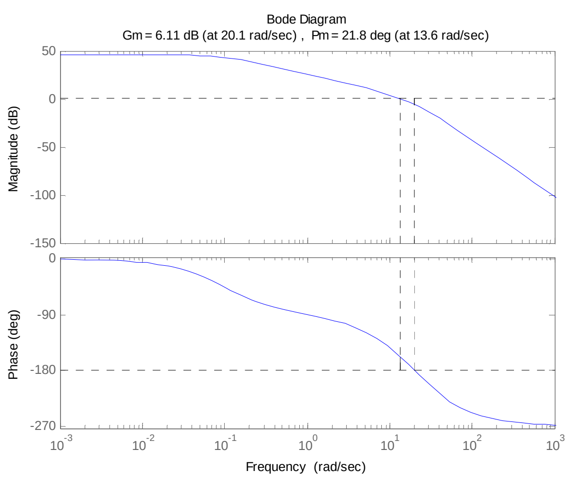
Idea: Design $K(s)$ such that $L(s) = \frac{\omega_c}{s}$ or

$$K(s) = G^{-1}(s)\,\frac{\omega_c}{s}$$

Might not be possible if:

- $G(s)$ has RHP-zeros (internal stability)
- $G(s)$ has pole excess of 2 or larger

Example 2.9: $G(s) = \frac{200}{10s + 1} \frac{1}{(0.05s + 1)^2}$



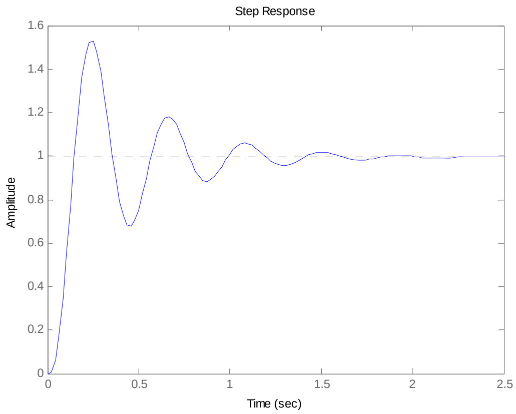
Design requirement:

- Rise time < 0.3 s
- overshoot < 5%

← Bode Plot of $G(s)$

Bandwidth requirement: $0.1 \approx e^{-\omega_{c\,min} \cdot 0.3}$

$$\omega_{c\,min} \approx \frac{\ln 10}{0.3} \approx 7.7 \text{ rad/s}$$

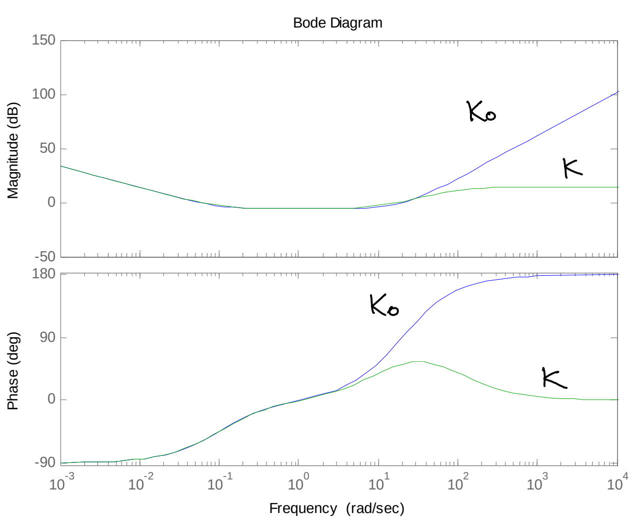


Step response: Fast enough, but too much overshoot. We try to apply inverse-model based design.

$K_0(s) = G^{-1}(s) \dfrac{\omega_c}{s}$, take $\omega_c = 10$, so

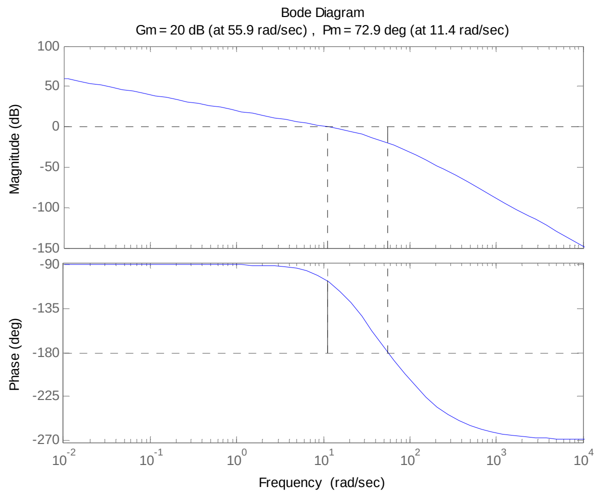
$K_0(s) = \dfrac{10}{s} \cdot \dfrac{10s+1}{200} \cdot (0.05s+1)^2$ ← not proper!

Approximate with $K(s) = \dfrac{10}{s} \cdot \dfrac{10s+1}{200} \cdot \dfrac{(0.1s+1)}{(0.01s+1)}$

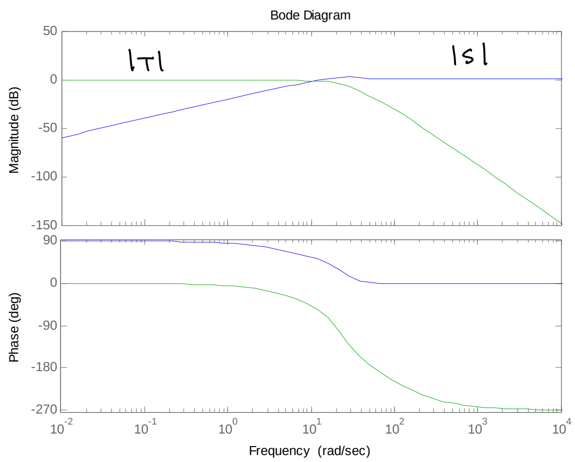


Bode Plots of Ko and K

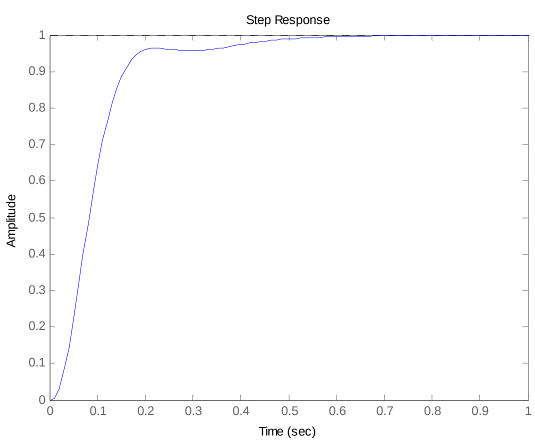
Close loop performance:



← Bode Plot of $G(s)K(s)$
Improved stability margin
good bandwidth



← good peak magnitudes M_S and M_T



← good step response

Loop shaping for disturbance rejection

Transfer function: $e = S\,G_d \cdot d$, thus if $S(s) = \frac{s}{s+\omega_c}$,

$$e = \frac{s}{s+\omega_c} G_d \cdot d \leftarrow \text{might not be good}$$

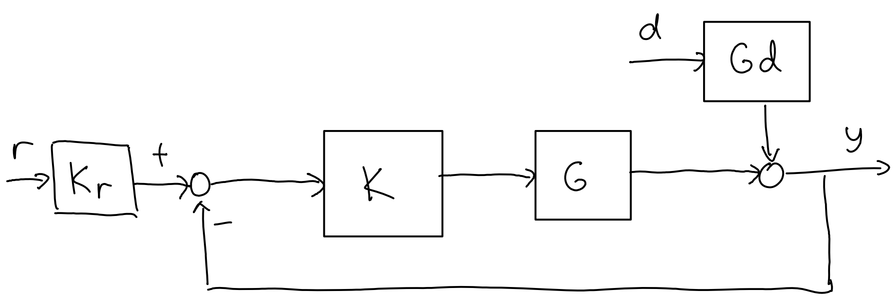
Ideal design: $|S(j\omega)||G_d(j\omega)| < 1, \forall \omega$, or

$|I + L(j\omega)| > |G_d(j\omega)|$, or within the bandwidth of G_d,

$|L| > |G_d|$ or equivalently $|K| > |G^{-1} G_d|$

- Study Example 2.10 → sometime it is impossible to attain good tracking and good disturbance at the same time.

Solution: Two degrees of freedom design



$$e = -S K_r r + S G_d d$$

↑ use K_r to compensate