Generally speaking, close-loop stability (i.e. from the poles of $S$ or $T$) is not sufficient for determining the stability of the system.

**Internal Stability:**

\[
Y = dy + G(U + du)
\]

\[
Y = dy + G(K(r-y) + du)
\]

\[
(1+GK)Y = dy + 6Kr + Gdu
\]

\[
Y = (I+6K)^{-1}dy + (I+GK)^{-1}6Kr + (I+6K)Gdu
\]

\[
U = K(r-y)
\]

\[
U = K(r - G(U + du) + dy)
\]

\[
(1+KG)U = Kr - KGdu + Kdy
\]

\[
U = (1+KG)^{-1}Kr - (I+KG)Gdu + (I+KG)^{-1}Kdy
\]

All of these transfer functions must be stable.

For SISO systems: 
\[
\frac{K}{1+6K}, \frac{6K}{1+6K}, \frac{1}{1+6K}, \frac{G}{1+6K}
\]

must be stable.

Suppose: $G(s) = \frac{A(s)}{B(s)}$; $K(s) = \frac{C(s)}{D(s)}$.

And $A$ and $B$ do not share common factors.

$C$ and $D$
- Suppose that the controller cancels a RHP zero of the plant with a RHP pole:

\[ A(s) = (s - z) \hat{A}(s) \]
\[ D(s) = (s - z) \hat{D}(s) \]

\[ S = \frac{1}{1+6kC} = \frac{1}{1 + \frac{AC}{BD}} = \frac{BD}{BD + AC} = \frac{(s-z)\hat{B}D}{(s-z)\hat{B}D + (s-z)\hat{A}C} \]

\[ \text{no unstable pole } \Rightarrow \hat{B}D \]
\[ \frac{BD + \hat{A}C}{\hat{B}D + \hat{A}C} \]

However:

\[ K = \frac{\hat{S}}{D} = \frac{\hat{S}}{BD + AC} = \frac{1}{AC + BD} = \frac{\hat{B}C}{(s-z)\hat{A}C + BD} \]

\[ \text{unstable pole} \]

Theorem: If there is no unstable pole-zero cancellation, then it is sufficient to check the stability of just one of the four transfer functions.

• If \( G(s) \) has a RHP zero at \( z \), then \( L, T \), and \( SG \) also have a RHP zero at \( z \).

\[ A(s) = (s-z)\hat{A}(s) \rightarrow L(s) = (s-z)\frac{\hat{A}C}{BD} \rightarrow L(s) = (s-z)\frac{\hat{A}C}{BD} \]

\[ T = \frac{L}{1+L} = \frac{(s-z)\hat{C}}{1+(s-z)\hat{C}} \]

\[ SG = \frac{G}{1+6kC} = \frac{A}{1 + \frac{AC}{BD}} = \frac{AD}{AC + BD} \]

\[ = \frac{(s-z)\hat{A}D}{(s-z)\hat{A}C + BD} \]

but \( BD \) cannot have \((s-z)\) as a factor, so \( SG \) has \( z \) as a zero.
If \( G \) has a RHP pole at \( p \) then
- \( L \) will have a RHP at \( p \)
- \( S \) and \( K_S \) will have a RHP zero at \( p \) (show it!)