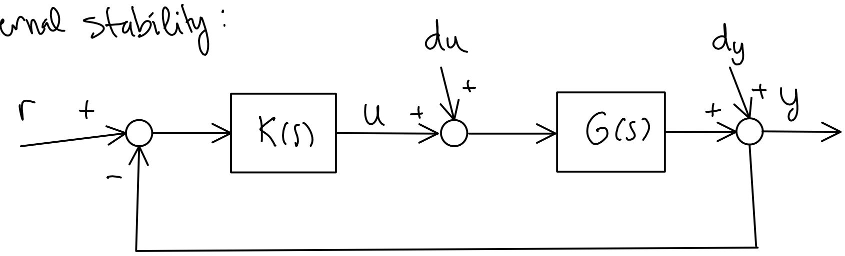
Internal Stability (Section 4.7)

Generally speaking, close loop stability (i.e from the poles of S or T) is not sufficient for determining the stability of the systems.

Internal stability:



$$Y = d_y + G(U + d_u)$$
$$Y = d_y + G(K(r-y) + d_u)$$

$$(I+GK)Y = d_y + GKr + Gd_u$$
$$Y = (I+GK)^{-1}d_y + (I+GK)^{-1}GKr + (I+GK)G\,d_u$$

$$U = K(r - Y)$$
$$= K(r - G(U+d_u) + d_y)$$
$$(I+KG)\,U = Kr - KG\,d_u + Kd_y$$
$$U = (I+KG)^{-1}Kr - (I+KG)^{-1}KG\,d_u + (I+KG)^{-1}Kd_y$$

All of these transfer functions must be stable!

For SISO systems: $\dfrac{K}{1+GK}$, $\dfrac{GK}{1+GK}$, $\dfrac{1}{1+GK}$, $\dfrac{G}{1+GK}$ must be stable

Suppose: $G(s) = \dfrac{A(s)}{B(s)}$; $K(s) = \dfrac{C(s)}{D(s)}$

A and B do not share common factors

C and D —— " ————

- Suppose that the controller cancels a RHP zero of the plant with a RHP pole:

$$A(s) = (s-z)\tilde{A}(s)$$
$$D(s) = (s-z)\tilde{D}(s)$$

$$S = \frac{1}{1+GK} = \frac{1}{1+\frac{AC}{BD}} = \frac{BD}{BD+AC} = \frac{(s-z)B\tilde{D}}{(s-z)B\tilde{D}+(s-z)\tilde{A}C}$$

no unstable pole $\rightarrow$ $= \dfrac{B\tilde{D}}{B\tilde{D}+\tilde{A}C}$

However: $\dfrac{K}{1+GK} = \dfrac{\frac{C}{D}}{1+\frac{AC}{BD}} = \dfrac{BC}{AC+BD} = \dfrac{1}{(s-z)}\,\dfrac{BC}{\tilde{A}C+B\tilde{D}}$

unstable pole

Theorem: If there is no unstable pole-zero cancellation, then it is sufficient to check the stability of just one of the four transfer functions.

* If $G(s)$ has a RHP zero at z then L, T, and SG also have a RHP zero at z.

$A(s) = (s-z)\tilde{A}(s) \rightarrow L(s) = (s-z)\dfrac{\tilde{A}C}{BD} \rightarrow L(s) = (s-z)\tilde{L}(s)$

$$T = \frac{L}{1+L} = \frac{(s-z)\tilde{L}}{1+(s-z)\tilde{L}}$$

$$SG = \frac{G}{1+GK} = \frac{\frac{A}{B}}{1+\frac{AC}{BD}} = \frac{AD}{AC+BD}$$

$$= \frac{(s-z)\tilde{A}D}{(s-z)\tilde{A}C+BD}$$

but BD cannot have $(s-z)$ as a factor, so SG has z as a zero.

If G has a RHP pole at p then

- L will have a RHP at p
- S and KS will have a RHP zero at p (show it!)