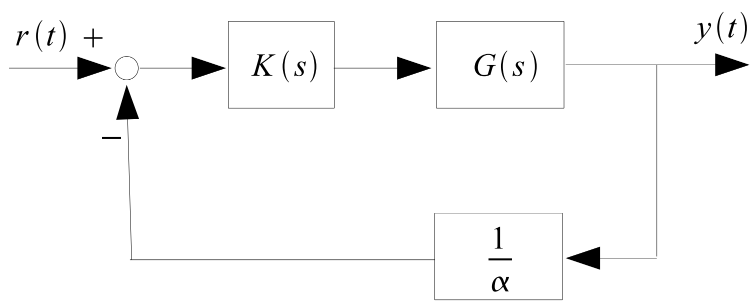
ECSE 6460: Multivariable Control Systems

Homework set 1. Due date: 18 September 2009

Points: Problem 1 = 10 pts, Problem 2 = 5+10+10+10 pts, Problem 3 = 10+20 pts, Problem 4 = 25 pts

Problem 1. Consider a SISO feedback system, with plant transfer function $G(s)$ and feedback controller $K(s)$. Prove that the sensitivity function $S(s) = \frac{1}{1+G(s)K(s)}$ and the complementary sensitivity function $T(s) = \frac{G(s)K(s)}{1+G(s)K(s)}$ have the same poles.

Problem 2. (Feedback amplifier from page 25) Consider the following block diagram of a feedback amplifier:



The plant,

$$G(s) = \frac{1000}{s+1}$$

is a high-gain amplifier. The goal of the feedback system is to achieve α amplification of the signal $r(t)$, or

$$y(t) = \alpha \cdot r(t).$$

Define the error signal $e(t) = r(t) - \frac{1}{\alpha}y(t)$.
(a) Compute the transfer function from r to e.
(b) Assume that the controller $K(s)$ is absent (set $K(s) = 1$). Use MATLAB to simulate the the response of the system for $r(t) = \sin t$. For each value $\alpha = 10, 50$, and 200, plot $r(t)$ and $\frac{y(t)}{\alpha}$ in one graph (one graph for each α). Explain what you observe in the context of high gain feedback.
(c) Assume that the controller $K(s)$ is absent (set $K(s) = 1$) and set $\alpha = 10$. Use MATLAB to simulate the the response of the system for $r(t) = \sin \omega t$. For each value $\omega = 1, 10$, and 100, plot $r(t)$ and $\frac{y(t)}{\alpha}$ in one graph. Explain what

you observe in the context of the bandwidth of the loop transfer function (you might want to include a Bode Plot).
(d) Design a controller of the form

$$K(s) = 1 + \frac{1}{\tau s},$$

by choosing an appropriate value for τ, such that the performance of the feedback amplifier for $\alpha = 10$ and $\omega = 100$ is improved. Plot the response of the system with your designed controller, and motivate your choice of τ (relate it with the bandwidth of the loop transfer function).

Problem 3. Consider a feedback control system with the plant transfer function

$$G(s) = \frac{6}{(s+2)(s-3)},$$

and the controller

$$K(s) = k \cdot \frac{s+1}{s+3},$$

with tunable gain k.
(a) Determine the stability of the close loop system if $k = 1$.
(b) Determine the condition on the gain k such that the close loop system is stable.

Problem 4. Use inverse based controller design (Section 2.6.3) to design a feedback controller for the plant

$$G(s) = \frac{s+1}{s(s+2)}$$

to achieve the following objectives: the settling time is less than 0.1 sec and overshoot is less than 5%. Explain your design and demonstrate its performance in MATLAB simulation.