Problem 1. Given a state-space representation of a system with disturbance
\[ \dot{x} = Ax + Bu + Gd, \]
\[ y = Cx, \]
with
\[
A = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix},
B = \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix},
G = \begin{bmatrix}
1 \\
1 \\
1 \\
1 \\
1
\end{bmatrix},
C = \begin{bmatrix}
1 & 0 & 0 & -1 & 0 \\
1 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & -1
\end{bmatrix}.
\]

(a) Compute the largest controlled invariant subspace of ker $C$.
(b) Can the Disturbance Decoupling Problem (DDP) be solved for this system? If so, compute the state feedback that solves it.

Problem 2. Consider the following input-state system:
\[ \dot{x} = Ax + Bu, \]
\[ A = \begin{bmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
0 & 0 & a_{33}
\end{bmatrix},
B = \begin{bmatrix}
b_1 \\
b_2 \\
0
\end{bmatrix}.
\]
Assume that all the coefficients above are nonzero, unless explicitly stated.

(a) Determine if $(A, B)$ is controllable.
(b) Show that $a_{33}$ is one of the open loop poles.
(c) Prove that for any linear state feedback $u = -Kx$, $a_{33}$ is one of the close loop poles.
(d) Based on (a) - (c), if given $(A, B)$, $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$ that is not controllable, describe a procedure to find all the open loop poles that cannot be moved by linear state feedback.