**ECSE 6460: Multivariable Control Systems**

**Homework set 4. Due date: 24 November 2009**

**Points:** Problem 1 = 15+15+10 pts, Problem 2 = 10+15+10+25 pts

**Problem 1.** Consider the following plant

\[ G(s) = \begin{bmatrix} \frac{s-1}{s-2} & -\frac{0.1s+1}{s-2} \\ \frac{0.1s+1}{s-1} & 1 \end{bmatrix}. \]

(a) Determine all the poles and zeros of the plant, and compute their input and output directions

(b) Determine the bound for \( \|S\|_\infty \) and \( \|T\|_\infty \) using Theorem 6.1 in the textbook.

(c) Determine the bound for \( \|S\|_\infty \) and \( \|T\|_\infty \) using formula (6.11) in the textbook.

**Problem 2.** (From Example 6.3) Consider the following plant

\[ G_\alpha(s) = \begin{bmatrix} 1 & 0 \\ \frac{1}{s-3} & 0 \end{bmatrix} \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \frac{s-3}{0.1s+1} & 0 \\ 0 & \frac{s+2}{0.1s+1} \end{bmatrix}, \]

(a) Show (analytically, not through numerical calculation) that the poles and zeros of the plant do not depend on \( \alpha \).

(b) For four values of \( \alpha \) (0°, 30°, 60°, and 90°) compute the output directions of any RHP poles and zeros.

(c) Compute the bound for \( \|S\|_\infty \) and \( \|T\|_\infty \) for each of values of \( \alpha \).

(d) Use \( H_\infty \) S/T mixed synthesis to construct a controller for each of values of \( \alpha \). Compare the \( \|S\|_\infty \) and \( \|T\|_\infty \) that you obtain for each case and the bounds that you computed in part c (i.e. compile a table like the one on page 227). Comment on any overshoot and undershoot that you observe in the step reference tracking performance. Is there any value of \( \alpha \) that results in no overshoot or undershoot in one of the outputs? Explain the result. Note: Remember that \( S \) and \( T \) are matrices and

\[ \|S\|_\infty := \sup_\omega \sigma_{\max}(S(j\omega)), \]

\[ \|T\|_\infty := \sup_\omega \sigma_{\max}(T(j\omega)). \]