ECSE 6460: Multivariable Control Systems

Homework set 4. Due date: 24 November 2009

Points: Problem 1 = 15+15+10 pts, Problem 2 = 10+15+10+25 pts

Problem 1. Consider the following plant

$$G(s) = \begin{bmatrix} \frac{s-1}{s-2} & -\frac{0.1s+1}{s-2} \\ \frac{s-1}{0.1s+1} & 1 \end{bmatrix}.$$

(a) Determine all the poles and zeros of the plant, and compute their input and output directions
(b) Determine the bound for $\|S\|_\infty$ and $\|T\|_\infty$ using Theorem 6.1 in the textbook.
(c) Determine the bound for $\|S\|_\infty$ and $\|T\|_\infty$ using formula (6.11) in the textbook.

Problem 2. (From Example 6.3) Consider the following plant

$$G_\alpha(s) = \begin{bmatrix} \frac{1}{s-2} & 0 \\ 0 & \frac{1}{s+3} \end{bmatrix} \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \frac{s-3}{0.1s+1} & 0 \\ 0 & \frac{s+2}{0.1s+1} \end{bmatrix},$$

(a) Show (analytically, not through numerical calculation) that the poles and zeros of the plant do not depend on α.
(b) For four values of α (0°, 30°, 60°, and 90°) compute the output directions of any RHP poles and zeros.
(c) Compute the bound for $\|S\|_\infty$ and $\|T\|_\infty$ for each of values of α.
(d) Use H_∞ S/T mixed synthesis to construct a controller for each of values of α. Compare the $\|S\|_\infty$ and $\|T\|_\infty$ that you obtain for each case and the bounds that you computed in part c (i.e. compile a table like the one on page 227). Comment on any overshoot and undershoot that you observe in the step reference tracking performance. Is there any value of α that results in no overshoot or undershoot in one of the outputs? Explain the result. Note: Remember that S and T are matrices and

$$\|S\|_\infty := \sup_\omega \sigma_{\max}(S(j\omega)),$$
$$\|T\|_\infty := \sup_\omega \sigma_{\max}(T(j\omega)).$$