Nonlinear Pricing for Social Optimality of PEV Charging Under Uncertain User Preferences

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***Abstract*— In this paper, we analyze a framework of charging Plug-in Electric Vehicles (PEVs) where the electric utility (or aggregator) sets time-dependent prices for charging, and the PEVs choose their charging profiles so as to minimize their individual charging costs (maximize individual profits). We show that there exists pricing policies that results in social optimality (i.e., minimize the network-wide charging cost, or maximize the total economic surplus) under individually rational (selfish) decision-making by the PEVs. The pricing policy is non-linear and can differ across PEVs, and takes into account the uncertainty in the user (PEV-owner) charging preferences which are in general not known to the utility, but can possibly be stochastically estimated (predicted) through observations over time. We evaluate the proposed pricing policy through simulations, in terms of the mean and variance of the total electric load on a sample distribution network.**

I. Introduction

In order to reduce dependency on petroleum, the cost of the fuel consumption, greenhouse gas emissions, and at the same time increase vehicle engine performance efficiency, Plugged-In Electric Vehicles (PEVs) have been getting popular in recent years [1]. Based on the nationwide survey data [2], [3], the average US household load in 2009 was 1.3 KW, while the Level 1 PEV charging uses 1.96KW and Level 2 PEV charging uses 7.2KW of extra power load.

Therefore, if every household owns just one PEV in the near future, the peak demand of the grid load from charging the PEVs can increase the peak load by a factor between 2.5 to 6.5 times of the current peak load. Widespread use of PEVs is however likely to increase the average electricity consumption significantly, which would then become a major component of an average household energy bill. Additionally, it would generate very bursty demand patterns, as the PEVs may prefer to charge all at the same times. Since cost of electricity generation and supply have a strong dependence on the peak-to-average ratio in energy usage, more bursty demand for electricity is likely to increase the electricity prices significantly. Efficient management of this excess demand (due to PEV charging) is important not only for reduced cost to consumers, but also for efficient operation and stability of the grid.

Coordination of PEV charging demand, aimed at spreading them over in time or scheduling them at times when the grid is under-loaded, can be achieved in different ways. In this paper, we study a price-driven framework where pricing signals are used to induce users (PEV owners) to schedule charging at times when the grid is expected to be light-loaded. In this framework, the utility (aggregator) publishes time-dependent price charts (functions) for PEV charging, and the PEVs respond to that by scheduling their charging to minimize their charging cost (maximize their individual profit functions) while satisfying their individual requirements. Our goal is to design the pricing functions (which will in general be time and user-dependent) to be socially optimal, i.e., minimize the total energy distribution cost, or maximize the total economic surplus.

The question of coordinated PEV charging has been approached from different perspectives including game theory [4] [5], gradient optimization [6], [7], stochastic optimization [8], sequential quadratic optimization [9], [10], dynamic programming [11], and various other methods [1], [12], [13], [14]. While our overall goal is similar to that considered in the prior work mentioned above, i.e. minimizing the load variance in the distribution network, our work differs from existing work in the following aspects. Firstly, unlike much of the prior work, we consider a price-induced mechanism for PEV charging, posing it in a game-theoretic framework where the charging decisions are made by selfish users (PEV owners) minimizing their costs (given the prices). While [4] addresses the coordinated PEV scheduling question as a non-cooperative game, price-driven scheduling is not taken into account. While [5] explicitly considers price-driven scheduling, the model is significantly different from ours, as [5] poses the question as a Stackelberg game between the utility (leader) and PEVs (followers), where the utility sets a price to maximize its own revenue given selfish users; in our case, however, the pricing function is set so as to optimize the social cost (minimize cost of distribution, or maximize economic surplus).

In a related work, [15] proposes a linear pricing policy and shows that there exists a (fixed per-unit) pricing vector and a charging profile which is socially optimal. However, as we show in section II-D, a linear pricing policy may not lead to social optimality of all individually rational charging solutions. The nonlinear pricing we propose in this paper, would result in a charging profile that is necessarily socially optimal.

Some of the key differences of our work with the existing work in this context are as follows. Firstly, in contrast with most of the literature which considers linear pricing functions (fixed per-unit prices), we consider non-linear pricing functions, which is necessary in our case for attaining social optimality. Secondly, unlike many of the existing price-driven scheduling models (in PEV as well as other

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contexts) we take into consideration differences in timing constraints/preferences across users, which necessitates user-specific pricing functions. Furthermore, we assume that these constraints/preferences may not be generally known to the price-setter (the electric utility in our case), but could be estimated (statistically) possibly through historical observations of user behavior. This incompleteness of information is captured in terms of user "types", as typically done in a Bayesian game-theoretic framework. The framework and results in this paper can be viewed as an extension or generalization of those in our earlier work [16] which considers a similar price-driven PEV charging question but in a deterministic (complete information) setting where user's charging constraints are assumed to be known to the utility, or learned through an iterative game played between the utility and the PEV owners ahead of the charging period.

The model that we consider in this paper represents the reality better, where the charging constraints/preferences on a particular day may not even be known in advance to the users (PEV owners) themselves, not to mention the electric utility. The utility may however have statistical estimates of the charging constraints and preferences of users, obtained through observing the charging behavior of individual users over time, which can then be utilized in optimizing the charging schedules (indirectly, through the design of appropriate pricing functions) in a stochastic sense. The model and results considered in this paper should be viewed in this context, and can be used by the electric utility in setting prices for PEV charging in advance (say, a month, day, or even hours ahead) based on its estimates (predictions) of the user preferences (behavior). It is worth noting here that our proposed pricing policy can be viewed as Walrasian pricing [17] applied to the PEV charging question, extended to consider uncertain user preferences (incomplete information about users' charging constraints/behavior) through Bayesian game-theoretic modeling.

The paper is structured as follows. We describe the system model, pricing policy and social optimality analysis of the proposed pricing policy in Section II. In Section III, we extend our framework and results to include user satisfaction functions. In Section IV, we evaluate the proposed policy through simulations, and conclude in Section V.

II. Pricing PEV Charging for Social Optimality

In this section, we describe the system model, and present our pricing policy which computes certain time-dependent PEV-specific pricing functions based on available statistics on the charging preferences of individual PEVs. We show that under the proposed pricing policy, individually rational charging profile choices by selfish PEV owners results in social optimality, i.e., minimizes the energy distribution cost in the grid.

A. System Model

We model a system where an electric utility (or an aggregator) computes a price of charging for K plug-in electric vehicles (indexed $1, ..., K$) on a distribution network, where the price is to be used over a period of T time units, which are indexed as $1, ..., T$. Let $D(t)$ denote the base load (aggregated non-PEV load) over the entire distribution network at time t, for all t.

The utility has a *prior belief* about the possible preferences of the different users (PEV owners) which are captured as different *types* as commonly done in a Bayesian game theoretic framework [18]. A type of PEV is defined based on the charging start time, finish time, total amount of charging requested, and also other charging constraints such as the maximum charging rate at each time slot. We denote a *PEV k is of type θ_k*, if the owner of PEV k decides to charge PEV k based on the constraints defined in type θ_k. Let Θ_k denote the set of all possible types of PEV k. Let t^s_{k,θ_k} and t^f_{k,θ_k} denote the charging start time and finish time of PEV k of type θ_k and let $T_{k,\theta_k} = [t^s_{k,\theta_k}, t^f_{k,\theta_k}]$ denote the charging interval of PEV k of type θ_k.

Let $p_{k,\theta_k}(t)$ denote the charging rate of PEV k of type θ_k, explained as a function of time. We assume that the charging rate $p_{k,\theta_k}(t)$ is constant during time unit t, and therefore PEV k consumes $p_{k,\theta_k}(t)$ amount of energy during time unit t. Let $\mathbf{p}_{k,\theta_k} = (p_{k,\theta_k}(1), ..., p_{k,\theta_k}(T))$ denote the charging profile of PEV k of type θ_k. Note that in our framework, the charging profile is determined by the PEVs, responding to the charging prices set by the utility, in a way that we will discuss in this section. Let $\mathbf{p}_k = (\mathbf{p}_{k,1}, ..., \mathbf{p}_{k,|\Theta_k|})$ denote the total charging profile of PEV k and $\mathbf{p} = (\mathbf{p}_1, \mathbf{p}_2, ..., \mathbf{p}_K)$ denote the charging profile of all PEVs charging in different types. Let $\mathbf{p}_{-k}$ denote the total charging profile of all PEVs, excluding that of PEV k. For PEV k of type θ_k, the charging rate should be within a range, due to constraints on the PEV battery and limits of charging equipments that PEV has access to at time t:

$$0 \leq p_{k,\theta_k}(t) \leq p^{max}_{k,\theta_k}(t), \qquad t \in T_{k,\theta_k}. \tag{1}$$

Obviously, $p_{k,\theta_k}(t) = 0$ for $t \notin T_{k,\theta_k}$. Let us define U_{k,θ_k} as the total energy PEV k of type θ_k requests to get charged in T_{k,θ_k}. Hence:

$$\sum_{t \in T_{k,\theta_k}} p_{k,\theta_k}(t) = U_{k,\theta_k}, \qquad \forall k, \theta_k \tag{2}$$

Let $\mathcal{D}_{k,\theta_k} = \{\mathbf{p}_{k,\theta_k} | \sum_{t \in T_{k,\theta_k}} p_{k,\theta_k}(t) = U_{k,\theta_k}, 0 \leq p_{k,\theta_k}(t) \leq p^{max}_{k,\theta_k}(t)\}$ denote the set of all feasible charging profiles of PEV k of type θ_k. Let $\mathcal{D}_k = \mathcal{D}_{k,1} \times ...\mathcal{D}_{k,|\Theta_k|}$ denote the set of all possible charging profiles of all types of PEV k and $\mathcal{D} = \mathcal{D}_1 \times ... \times \mathcal{D}_K$ denote all the feasible charging profiles of all the PEVs charging in all different types.

The probability that PEV k is of type θ_k is denoted as ϕ_{k,θ_k}. We assume that the utility has estimates of the probability ϕ_{k,θ_k} calculated based on historical observation of the individual user (PEV owner) charging behavior. Obviously, $\sum_{\theta_k=1}^{|\Theta_k|} \phi_{k,\theta_k} = 1$, for all k. Let $\phi(\theta_1, ..., \theta_K)$ denote the joint probability that PEVs $(1, ..., K)$ are of types $(\theta_1, ..., \theta_K)$, respectively. Let us denote $\phi_{-k}(\theta_1, ..., \theta_K)$ as the joint probability that PEVs $(1, ..., k-1, k+1, ..., K)$ are of types $(\theta_1, ..., \theta_{k-1}, \theta_{k+1}, ..., \theta_K)$, respectively. We assume that each PEV determines its charging type independent from other PEVs in the grid, thus we have: $\phi(\theta_1, ..., \theta_K) = \prod_{k=1}^{K} \phi_{k,\theta_k}$ and $\phi_{-k}(\theta_1, ..., \theta_K) = \prod_{i=1, i\neq k}^{K} \phi_{k,\theta_k}$.

B. Social Optimality

The utility minimizes the social cost function defined as the expected value of a convex function of the total load in the grid as in the following:

$$\mathcal{L}(\mathbf{p}) = E_\phi \left(\sum_{t=1}^{T} \mathcal{V} \left(D(t) + \sum_{k=1}^{K} p_{k,\theta_k}(t) \right) \right)$$
$$= \sum_{t=1}^{T} \sum_{\theta_1=1}^{|\Theta_1|} ... \sum_{\theta_K=1}^{|\Theta_K|} \mathcal{V} \left(D(t) + \sum_{k=1}^{K} p_{k,\theta_k}(t) \right) \phi(\theta_1, ..., \theta_K), \tag{3}$$

where $\mathcal{V}(x)$ is a strictly convex function of x. For the case that the utility minimizes the variance of the total load, $\mathcal{V}(x)$ is set to $\mathcal{V}(x) = x^2$.

C. Individual Rationality

Let $\psi_k^t(x_k^t)$ denote the pricing function determined by the utility for PEV k to charge the x_k^t amount of energy at time t. As PEV k does not pay for the unused electricity, we have $\psi_k^t(0) = 0$. Let us denote $\mathbf{\Psi}_k(\mathbf{x}_k) = (\psi_k^1(x_k^1), ..., \psi_k^T(x_k^T))$ as the pricing policy for PEV k. Let vector $\mathbf{\Psi}(\mathbf{x}) = (\mathbf{\Psi}_1(\mathbf{x}_1), ..., \mathbf{\Psi}_K(\mathbf{x}_K))$ denote the pricing policy for all the PEVs in the grid. Given the pricing policy $\mathbf{\Psi}_k$, the charging cost of PEV k with charging profile $\mathbf{x}_k$, $\mathcal{J}_k(\mathbf{\Psi}_k, \mathbf{x}_k)$, is calculated as in the following:

$$\mathcal{J}_k(\mathbf{\Psi}_k, \mathbf{x}_k) = \sum_{t=1}^{T} \psi_k^t(x_k^t). \tag{4}$$

Assuming rational PEV owners, each PEV k minimizes its own charging cost of type θ_k, $\forall k, \theta_k$. Let the charging profile $\tilde{\mathbf{p}}_{k,\theta_k}$, minimizes the charging cost of PEV k of type θ_k. Hence:

$$\tilde{\mathbf{p}}_{k,\theta_k} = \arg \min_{\mathbf{p}_{k,\theta_k} \in \mathcal{D}_{k,\theta_k}} \mathcal{J}_k(\mathbf{\Psi}_k, \mathbf{p}_{k,\theta_k})$$
$$= \arg \min_{\mathbf{p}_{k,\theta_k} \in \mathcal{D}_{k,\theta_k}} \sum_{t \in T_{k,\theta_k}} \psi_k^t(p_{k,\theta_k}(t)). \tag{5}$$

D. Motivation for Nonlinear Pricing

Before we describe the utility's pricing policy, we motivate the use of nonlinear pricing by showing a simple example that a linear pricing function does not lead to the social optimal charging profile. Let the social cost function is the variance of the total load in the grid, i.e $\mathcal{V}(x) = x^2$. Let us assume that $T = 2$ hours and there is only one PEV 1 of only one type 1 charging amount of $U_{1,1} = 2$KWh in hours indexed as 1 and 2 with $p_{1,1}^{max}(t) = 2$ KW, for $t = 1, 2$. Let us assume $D(1) = 421$KWh and $D(2) = 420$KWh. As $D(1) > D(2)$, the utility proposes the linear pricing functions $\psi_1^1(x) = ax$ and $\psi_1^2(x) = bx$, with $a > b$, to prevent PEV 1 from increasing the variance of the total load in the grid. PEV 1 sets its charging profile of type 1 as $\mathbf{p}_{1,1} = (0, 2)$KW to minimize its charging cost. In this case, the total load in the grid is $l(1) = 421$KWh and $l(2) = 422$KWh. The mean value of the total load is 421.5 KWh and the variance of the total load is calculated as $0.5^2 + 0.5^2 = 0.5$ (KWh)^2. For the pricing vector $\mathbf{\Psi}^* = (ax, ax)$, the charging profile $\mathbf{p}^* = (0.5, 1.5)$ KW, is socially optimal (the variance of the total load is minimized to zero), however, under this proposed linear pricing policy, PEV 1 may choose many charging profiles that do not lead to the social optimality. This simple example illustrates that the linear pricing policy does not necessarily lead to the social optimally charging profile of the PEVs in the grid and there is a necessity to find a nonlinear pricing policy that forces PEVs to choose the social optimal charging profile. This urges us to explore beyond linear pricing policies.

E. Proposed Pricing Policy

Next we discuss the proposed (nonlinear, user-dependent) pricing policy. Let $\mathbf{p}^*$ denote the social optimum charging profile that minimizes $\mathcal{L}(\mathbf{p})$ in equation (3); Note that $\mathbf{p}^*$ can be computed in different ways such as the best response strategy based approach proposed in [16]. We propose the pricing function ${\psi^*}_k^t(x_k^t)$, $t \in [1, T]$, for each PEV k, as in the following:

$${\psi^*}_k^t(x_k^t) = E_{\phi_{-k}} \left(\mathcal{V} \left(D(t) + \sum_{i=1, i\neq k}^{K} p_{i,\theta_i}^*(t) + x_k^t \right) \right)$$
$$- E_{\phi_{-k}} \left(\mathcal{V} \left(D(t) + \sum_{i=1, i\neq k}^{K} p_{i,\theta_i}^*(t) \right) \right), \tag{6}$$

where x_k^t is the amount of energy that PEV k decides to charge at time slot t.

F. Analysis

We next show that the proposed pricing policy as derived in equation (6) leads to the socially optimal charging profile when selfish users choose their charging profiles to minimize their individual costs. Let us define the cost-minimizing socially optimal pricing policy as follows.

Definition 1: A pricing policy, $\mathbf{\Psi}$, is *cost-minimizing socially optimal*, if the charging profile that minimizes the individual cost function of each PEV k of type θ_k, under pricing policy $\mathbf{\Psi}$, is the socially optimal charging profile.

Lemma 1: Given the pricing policy $\mathbf{\Psi}_k^*$ as in equation (6), the optimal solution $\hat{\mathbf{p}}_{k,\theta_k} \in \mathcal{D}_{k,\theta_k}$ that minimizes the cost of PEV k of type θ_k, $\mathcal{J}_k(\mathbf{\Psi}_k^*, \mathbf{p}_{k,\theta_k})$ as defined in equation (5), is unique.

Proof: As $\mathcal{V}(x)$ is a strictly convex function of x, ${\psi^*}_k^t(x_k^t)$ is a strictly convex function of x_k^t. Therefore the cost function $\mathcal{J}_k(\mathbf{\Psi}_k^*, \mathbf{p}_{k,\theta_k}) = \sum_{t \in T_{k,\theta_k}} {\psi^*}_k^t(p_{k,\theta_k}(t))$, is strictly convex in $\mathcal{D}_{k,\theta_k}$, and there exists a unique charging profile $\tilde{\mathbf{p}}_{k,\theta_k} \in \mathcal{D}_{k,\theta_k}$ that minimizes $\mathcal{J}_k(\mathbf{\Psi}_k^*, \mathbf{p}_{k,\theta_k})$ in $\mathcal{D}_{k,\theta_k}$. ■

Theorem 1: The pricing policy $\mathbf{\Psi}^*$ defined as in equation (6), is cost minimizing socially optimal.

Proof: Let $\mathbf{p}^*$ is the socially optimal charging profile and let $\tilde{\mathbf{p}}$ denote the charging profile such that $\tilde{\mathbf{p}}_{k,\theta_k}$ minimizes the charging cost of PEV k of type θ_k under the proposed pricing policy defined as in equation (6), for all k, θ_k. Using equations (5) and (6), the charging profile $\tilde{\mathbf{p}}_{k,\theta_k}$ minimizes $\mathcal{J}_k(\mathbf{\Psi}^*, \mathbf{p}_{k,\theta_k})$ in $\mathcal{D}_{k,\theta_k}$ for each PEV k of type θ_k, as in the following:

$$\begin{aligned}\tilde{\mathbf{p}}_{k,\theta_k} &= \arg \min_{\mathbf{p}_{k,\theta_k} \in \mathcal{D}_{k,\theta_k}} \mathcal{J}_k(\mathbf{\Psi}_k^*(\mathbf{x}_k), \mathbf{p}_{k,\theta_k}) \\ &= \arg \min_{\mathbf{p}_{k,\theta_k} \in \mathcal{D}_{k,\theta_k}} \sum_{t=1}^{T} {\psi^*}_k^t(p_{k,\theta_k}(t)) \\ &= \arg \min_{\mathbf{p}_{k,\theta_k} \in \mathcal{D}_{k,\theta_k}} \sum_{t=1}^{T} E_{\phi_{-k}} \Bigg(\mathcal{V}\Bigg(D(t) \\ &\quad + \sum_{i=1, i\neq k}^{K} p^*_{i,\theta_i}(t) + p_{k,\theta_k}(t)\Bigg)\Bigg) \end{aligned} \quad (7)$$

Note that the second term in equation (6) is constant with respect to the optimizing variable vector, $\mathbf{p}_{k,\theta_k}$, and it is ignored in equation (7). Based on the assumption that each PEV chooses its charging type independently from other PEVs, the social cost function $\mathcal{L}(\mathbf{p}^*_{-k}, \mathbf{p}_k)$, defined as in equation (3), is written as in the following:

$$\begin{aligned}\mathcal{L}(\mathbf{p}^*_{-k}, \mathbf{p}_k) &= \sum_{t=1}^{T} E_{\phi_k} \Bigg[E_{\phi_{-k}} \Bigg(\mathcal{V}\Bigg(D(t) \\ &\quad + \sum_{i=1, i\neq k}^{K} p^*_{i,\theta_i}(t) + p_{k,\theta_k}(t)\Bigg)\Bigg)\Bigg] \\ &= \sum_{\theta_k=1}^{|\Theta_k|} \Bigg[\sum_{t=1}^{T} E_{\phi_{-k}} \Bigg(\mathcal{V}\Bigg(D(t) \\ &\quad + \sum_{i=1, i\neq k}^{K} p^*_{i,\theta_i}(t) + p_{k,\theta_k}(t)\Bigg)\Bigg)\Bigg] \phi_{k,\theta_k} \end{aligned} \quad (8)$$

Based on equation (7), the charging profile $\tilde{\mathbf{p}}_{k,\theta_k}$ minimizes each term in the last equation of (8) in $\mathcal{D}_{k,\theta_k}$. Therefore, the charging profile $\tilde{\mathbf{p}}_k$, minimizes $\mathcal{L}(\mathbf{p}^*_{-k}, \mathbf{p}_k)$ in $\mathcal{D}_k$. As $\mathcal{L}(\mathbf{p}^*_{-k}, \mathbf{p}_k)$ is strictly convex in $\mathbf{p}_k$, the charging profile $\tilde{\mathbf{p}}_k$ that minimizes $\mathcal{L}(\mathbf{p}^*_k, \mathbf{p}_k)$ in $\mathcal{D}_k$, is unique. Furthermore, $\mathbf{p}^*$ minimizes $\mathcal{L}(\mathbf{p})$ in $\mathbf{p} \in \mathcal{D}$. Therefore, we have:

$$\mathcal{L}(\mathbf{p}^*) = \mathcal{L}(\mathbf{p}^*_{-k}, \mathbf{p}^*_k) \leq \mathcal{L}(\mathbf{p}^*_{-k}, \tilde{\mathbf{p}}_k), \quad (9)$$

It shows that the unique minimizing charging profile $\tilde{\mathbf{p}}_k$ in $\mathcal{D}_k$ must be equal to $\mathbf{p}^*_k$. ■

Under the pricing policy in equation (6), individually rational (selfish) PEV k is forced to choose the unique socially optimal charging profile $\mathbf{p}^*_{k,\theta_k}$ in order to minimize its charging cost in each charging type $\theta_k \in \Theta_k$. Therefore, the proposed pricing mechanism minimizes the expected cost of the utility that is defined in equation (3).

G. Expected Net Income of the Utility

Let us assume that the social cost function, as defined in equation (3), represents the expected purchase cost of the electricity for the utility and the utility desires to maximize its expected net income. The expected purchase cost of electricity is calculated as in the following:

$$\begin{aligned}\mathcal{C}(\mathbf{p}) &= E_\phi \left(\sum_{t=1}^{T} \mathcal{V}\left(D(t) + \sum_{k=1}^{K} p_{k,\theta_k}(t)\right) - \mathcal{V}\left(D(t)\right)\right) \\ &= \mathcal{L}(\mathbf{p}) - \mathcal{L}(\mathbf{0}). \end{aligned} \quad (10)$$

Based on Theorem 1, the utility proposes the pricing policy $\mathbf{\Psi}^*$ as in equation (6) to the PEVs in the grid, to minimize its purchase cost. The expected revenue of the utility by selling energy to the PEVs is calculated as in the following:

$$\mathcal{R}(\mathbf{p}) = E_\phi \left(\sum_{k=1}^{K} \sum_{t=1}^{T} {\psi^*}_k^t(p_{k,\theta_k}(t))\right) \quad (11)$$

The expected net income of the utility is calculated as in the following:

$$\mathcal{I}(\mathbf{p}) = \mathcal{R}(\mathbf{p}) - \mathcal{C}(\mathbf{p}). \quad (12)$$

Let $\mathcal{D}^*$ denote the set of all socially optimal charging profiles. As $\mathcal{L}(\mathbf{p})$ is a convex function in the convex set $\mathcal{D}$, therefore $\mathcal{D}^*$ is also a convex set. From Theorem 1, all the charging profile $\mathbf{p}^* \in \mathcal{D}^*$ resulting from the proposed pricing policy $\mathbf{\Psi}^*$, minimize the electricity cost purchase, $\mathcal{C}(\mathbf{p})$. As $\mathcal{C}(\mathbf{p}^*)$ is minimized for all $\mathbf{p}^* \in \mathcal{D}^*$, the utility must choose a charging profile, $\hat{\mathbf{p}}^* \in \mathcal{D}^*$, that maximizes $\mathcal{R}(\mathbf{p}^*)$ over $\mathcal{D}^*$ in order to maximize its net income.

III. INCORPORATING USER SATISFACTION FUNCTION

So far, we have assumed that PEV k of type θ_k requests a fixed charging amount, U_{k,θ_k}. In general, the PEV will typically have some flexibility in terms of charging amount, and U_{k,θ_k} could only represent an upper bound of the energy requested over T_{k,θ_k}. In such a case, the value associated with a charging amount p_{k,θ_k} (for user k when it is of type θ_k) can be represented by a satisfaction function $\mathcal{S}(\mathbf{p}_{k,\theta_k})$, which we will assume to be of the form $\mathcal{S}(\mathbf{p}_{k,\theta_k}) = \mathcal{W}_{k,\theta_k}(\sum_{t\in T_{k,\theta_k}} p_{k,\theta_k}(t))$. We will assume that the function $\mathcal{W}(.)$ is strictly concave in its (scalar) argument, and is modeled/estimated by the utility.

Let $\tilde{\mathcal{D}}_{k,\theta_k} = \{\mathbf{p}_{k,\theta_k} | \sum_{t\in T_{k,\theta_k}} p_{k,\theta_k}(t) \leq U_{k,\theta_k}, 0 \leq p_{k,\theta_k}(t) \leq p^{max}_{k,\theta_k}(t)\}$ denote the set of charging profiles of PEV k of type θ_k. Let $\tilde{\mathcal{D}}_k = \tilde{\mathcal{D}}_{k,1} \times \tilde{\mathcal{D}}_{k,|\Theta|_k}$ denote the set of all charging profiles of PEV k; let $\tilde{\mathcal{D}} = \tilde{\mathcal{D}}_1 \times ... \times \tilde{\mathcal{D}}_K$ denote the set of all possible charging profiles of the PEVs.

In this case, the utility may be interested in maximizing the *economic surplus* defined as the total user satisfaction minus the total cost of energy, expressed as follows:

$$\mathcal{F}(\mathbf{p}) = E_{\phi}\left(\sum_{k=1}^{K}\mathcal{W}_{k,\theta_k}\left(\sum_{t\in T_{k,\theta_k}} p_{k,\theta_k}(t)\right)\right.$$
$$\left.-\sum_{t=1}^{T}\mathcal{V}\left(D(t)+\sum_{k=1}^{K}p_{k,\theta_k}(t)\right)\right). \quad (13)$$

Assuming individually rational (selfish) PEV owners, each PEV k maximizes it own charging profit of type $\theta_k, \forall k, \theta_k$. The charging profit of each PEV k of type θ_k, under the pricing policy $\mathbf{\Psi}_k$, is calculated as in the following:

$$\mathcal{U}_{k,\theta_k}\left(\mathbf{\Psi}_k, \mathbf{p}_{k,\theta_k}\right)$$
$$= \mathcal{W}_{k,\theta_k}\left(\sum_{t\in T_{k,\theta_k}} p_{k,\theta_k}(t)\right) - \sum_{t\in T_{k,\theta_k}} \psi_k^t(p_{k,\theta_k}(t)). \quad (14)$$

The expected profit of PEV k, under pricing policy $\mathbf{\Psi}(\mathbf{x})$, is calculated as in the following:

$$\bar{\mathcal{U}}_k(\Psi_k, \mathbf{p}_k) = E_{\phi_k}\left(\mathcal{U}_k(\mathbf{\Psi}_k, \mathbf{p}_{k,\theta_k})\right)$$
$$= \sum_{\theta_k=1}^{|\Theta_k|}\left(\mathcal{W}_{k,\theta_k}\left(\sum_{t\in T_{k,\theta_k}} p_{k,\theta_k}(t)\right)\right.$$
$$\left.-\sum_{t\in T_{k,\theta_k}} \psi_k^t(p_{k,\theta_k}(t))\right)\phi_{k,\theta_k}. \quad (15)$$

Let $\tilde{\mathbf{p}}_{k,\theta_k}$ maximizes $\mathcal{U}_k(\mathbf{\Psi}_k, \mathbf{p}_{k,\theta_k})$, for all $\theta_k \in \Theta_k$, therefore $\tilde{\mathbf{p}}_k$ also maximizes $\bar{\mathcal{U}}_k(\Psi_k, \mathbf{p}_k)$ in $\tilde{\mathcal{D}}_k$.

Our proposed pricing policy is the same as before, defined by equation (6). We next show that this pricing policy results in a socially optimal charging profile (i.e. one that maximizes (13)), when the users choose their charging profile to maximize their individual profits (defined by (14)). We first prove the uniqueness of the charging profile $\tilde{\mathbf{p}}_{k,\theta_k}$ chosen by PEV k of type θ_k, under the proposed pricing policy as in (6).

Lemma 2: The charging profile $\tilde{\mathbf{p}}_{k,\theta_k}$ that maximizes the profit function of PEV k of type θ_k, $\mathcal{U}_k(\Psi_k(\mathbf{x}_k), \mathbf{p}_k)$, under the pricing policy,$\mathbf{\Psi}^*$, as defined in equation (6), is unique.

Proof: Let us denote $Y = [0, U_{k,\theta_k}]$. Let $\mathcal{Z}(y) = \{\mathbf{p}_{k,\theta_k} \in \tilde{\mathcal{D}}_{k,\theta_k} | \sum_{t\in T_{k,\theta_k}} p_{k,\theta_k}(t) = y\}$, where $y \in Y$. Let us denote $g(y) = \min_{p_{k,\theta_k}\in\mathcal{Z}(y)} \sum_{t\in T_{k,\theta_k}} {\psi^*}_k^t(p_{k,\theta_k}(t))$. The objective function of the PEV k of type θ_k to maximize its profit is rewritten as: $\max_{y\in Y}\left[\mathcal{W}_{k,\theta_k}(y) - g(y)\right]$. As $\mathcal{Z}(y)$ is a convex set and ${\psi^*}_k^t(x_k^t)$ is a strictly convex function of x_k^t, therefore $g(y)$ is a convex function of $y \in Y$. As $\mathcal{W}_{k,\theta_k}$ is a strictly concave function, therefore $\mathcal{W}_{k,\theta_k}(y) - g(y)$ is a strictly concave function of $y \in Y$. Thus there exists a unique $y^* \in Y$ that maximizes the profit of PEV k of type θ_k. Based on Lemma 1, there exists a unique charging profile $\tilde{\mathbf{p}}_{k,\theta_k} \in \mathcal{Z}(y^*)$ that achieves $g(y^*)$ and therefore $\tilde{\mathbf{p}}_{k,\theta_k}$ uniquely maximizes the charging profit of PEV k of type θ_k. ■

Definition 2: A pricing policy, $\mathbf{\Psi}$, is *profit maximizing socially optimal*, if the charging profile that maximizes the individual profit function of each PEV k of type θ_k, under pricing policy $\mathbf{\Psi}$, is the socially optimal charging profile.

Theorem 2: The pricing policy $\mathbf{\Psi}^*$ defined as in equation (6), is profit maximizing socially optimal.

Proof: Given the uniqueness of $\tilde{\mathbf{p}}_{k,\theta_k}$ (Lemma 2), the proof of Theorem 2 can be obtained in the similar way as proof of Theorem 1, substituting $\mathcal{L}$ with $-\mathcal{F}$, $\mathcal{J}_{k,\theta_k}$ with $-\mathcal{U}_{k,\theta_k}$, $\bar{\mathcal{J}}_k$ with $-\bar{\mathcal{U}}_k$ and $\mathcal{D}_k$ with $\tilde{\mathcal{D}}_k$. ■

IV. Numerical Results

For the simulation study we implement and evaluate the performance of the pricing policy that is proposed in this paper for the cost minimizing social optimum criterion. In our simulation study we aim to minimize the variance of the total load in the grid as the social cost function, therefore we set $\mathcal{V}(x)$ as $\mathcal{V}(x) = x^2$ in equation (3). The hourly load demand data is obtained from [19]. We assume that there are $K = 60$ PEVs in the distribution network under consideration, each of them charging in its own specific type. We also assume that there are two types of PEV owners/users charging in the grid denoted as office charger (type O) and home charger (type H). Each user is either an office charger or a home charger. All the users charge using level 1 or Level 2 charger with maximum power rate $p_{k,\theta_k}^{max}(t) = 1.96$KW and $p_{k,\theta_k}^{max}(t) = 7.2$KW, respectively.

Each office charger is further categorized in two different sub-types: early office charger (type OE) and late office charger (type OL). An early office charger charges during 8:00 am to 4:00 pm and a late office charger charges during 10:00 am to 6:00 pm. Each home charger is also categorized in two different types: the early charger (type HE) and the late charger (type HL). An early home charger charges during 6:00 pm to 6:00 am and a late home charger charges during 8:00 pm to 8:00 am. The battery capacity of an office charger is $U_{k,\theta_k} = 7$KWh and a home charger charges up to a battery capacity of $U_{k,\theta_k} = 12$KWh.

In the simulation results demonstrated in Figures 1-2, we have 30 office charger users and 30 home charger users. An office charger PEV is considered an early charger with probability 0.6 and a late charger with probability 0.4. A home charger PEV is considered an early charger with probability 0.7 and a late charger with probability 0.3. A home charger charges at level 1 with probabity 0.6 and charges in level 2 with probability 0.4. An office charger charges in level 1 with probability 0.7 and charges in level 2 with probability 0.3. We assume that the utility knows about the charging level of the PEVs ahead of scheduling.

Figure 1 shows that on the average, load is distributed such that the total demand is flattened as much as possible. Figures 2 shows the coefficient of variation, that is defined as the ratio of the standard deviation to the mean value of the total load in the grid for different hours during the day. As it is seen in Figure 2, the maximum coefficient of variation is the relatively the small value of 0.0085. Two figures show that the proposed pricing policy is quite effective in distributing the PEV demand so that the overall variance of

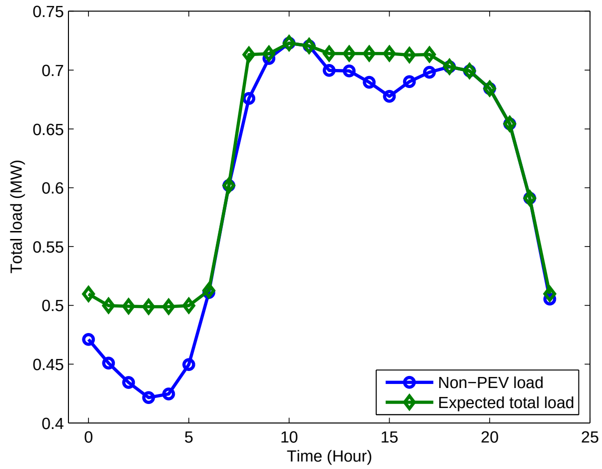

Fig. 1. Expected total load vs time in a daily 24 hour time zone of $K = 60$ PEVs charging in different types with the social cost function set as the variance of the total load.

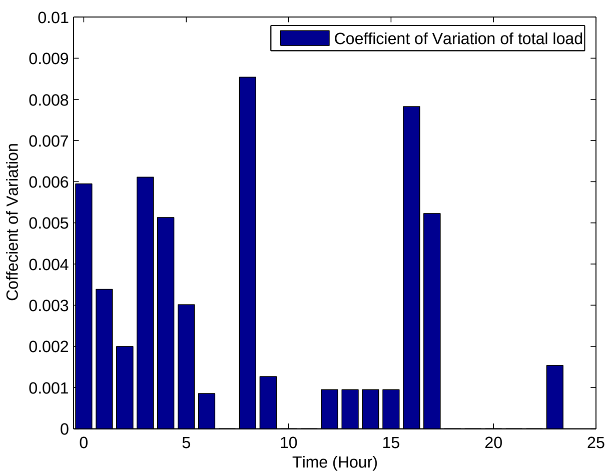

Fig. 2. The coefficient of variation of the total load vs time in a daily 24 hour time zone for the total number of $K = 60$ PEVs charging in different types with the social cost function set as the variance of the total load.

the total (PEV + non-PEV) load is minimized in a stochastic sense.

V. Conclusion

As mentioned earlier, the proposed policy can be viewed as a Bayesian extension of the widely studied Walrasian pricing [17], applied to our PEV charging model. The Bayesian modeling of the user preferences is necessitated due to fact that the exact user charging preferences/constraints may not be known a priori at price-setting time, but their statistics may be obtained through historical data analysis. Walrasian pricing not only matches demand with supply, but also results in social optimality in many contexts; therefore, the fact that our pricing policy results in social optimality should not come as a surprise. However, while much of the literature on Walrasian pricing considers only linear pricing functions, we argue for the necessity of nonlinear pricing functions in this case.

There are two research directions that are worth further investigation. Firstly, note that it may be possible to reduce the overall charging cost (or improve the economic surplus) even further by allowing the utility to set/change the prices dynamically, based on the current state of the system. Such dynamic pricing policies need to be modeled and analyzed in a multi-stage stochastic game-theoretic framework which will be considered in future work. Secondly, note that the socially optimal pricing policy that we proposed need not maximize the revenue of the electric utility, across all possible charging policies. Pricing that is revenue-optimal to the utility under individually cost-minimizing users may need to be considered in a Stackelberg game-theoretic framework; the social optimality (efficiency) question of Stackelberg pricing in our PEV charging model merits further investigation.

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