Threshold Activation Policies in a Random Sensing Environment

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Abstract

We consider the problem of how sensor nodes should be activated in so as to maximize a strictly concave utility function in a random sensing environment. Sensors are assumed to be energy-constrained, but rechargeable, and the energy discharge and recharge times are random. Under Markovian assumptions, we study the performance of threshold activation policies, for two different correlation models of the energy discharge and recharge processes. For both models, we show that the optimal threshold policy guarantees performance within factor of \( \frac{3}{4} \) of the optimum over all possible policies. We also comment on the effect of the correlation model on the performance of the threshold policies.

1 Introduction

Due to major technological innovations in recent years, development of tiny, low-cost sensor devices has become possible. Such sensor devices can be deployed in large numbers in different environments for monitoring and data gathering purposes [1]. These sensor devices, although cheap, are typically unreliable. Moreover, sensor devices are limited by battery energy. Therefore, a sensing device can remain powered on (and be sensing) only for a limited amount of time, until it runs out of battery energy [3]. In many scenarios, sensors can be rechargeable, but recharging is often a very slow process. Thus, the rate of recharging could be significantly less than the rate of energy depletion during the sensing period. As a result, a sensor could need to spend most of its lifetime in the “off” state, when it is not sensing, but recharging. These factors motivate redundant deployment of sensors to cover the area of interest. Each sensor being unreliable, sensing reliability increases if more number of sensors are sensing the same area at the same time. If larger number of sensors are deployed, it is likely that more number of these sensors would remain charged (and hence can be used for sensing) at any given time. Thus the overall system performance would typically improve (possibly with diminishing returns) with a more redundant deployment of sensors.

We consider the scenario mentioned above, where multiple sensors have been deployed to cover the same area. We assume that sensor nodes involved in sensing get discharged after a certain duration of time, and need to be recharged till they can be start sensing again. We consider the decision problem of when the recharged sensors should be activated (i.e., switched on) so as to maximize the long-term utility of the system. As is the case in reality, we assume that the discharge and recharge times of the sensors are random. We consider two extreme correlation models of the discharge/recharge times of the different sensors: one in which these times are highly correlated, and the other in which these times are independent of one another. It is worth noting here that our problem formulation is novel, and formalizes a practically important problem which has not yet received its due attention. If the discharge/recharge times are exponential, and certain other additional assumptions hold, then our problem can be posed as a Markov decision problem. Obtaining...
the optimal policy for this decision problem is computationally difficult, and this motivates us to look at the class of threshold decision policies. Threshold policies yield closed-form expressions, and the optimal threshold policy can be computed efficiently. We prove that the performance of the optimal threshold policy is very close to the best achievable performance. Therefore, consideration of threshold policies allows us to obtain near-optimal policies to our complex decision problem in an efficient manner.

Our main technical contributions are as follows. Under Markovian assumptions, we study the performance of threshold based decision policies for this problem. We show that the time-average utility of the optimal threshold policy is within a factor of \( \frac{3}{4} \) of the best possible performance, for both correlation models. Moreover, we show that correlation of the discharge/recharge times of the sensors degrades performance at all threshold values. We also complement our theoretical results with numerical studies.

2 Formulation

2.1 System Model and Assumptions

We consider a system of \( N \) sensors covering the same area. At any instant of time, each sensor could be in one of three states: i) active ii) passive, or iii) ready. In the active state, the sensors are powered on and are sensing. A sensor in the active state suffers a gradual depletion of battery energy, and enters the passive state when its battery is completely discharged. Sensors that are passive are powered off, and are simply recharging their batteries. When its battery is completely charged, the sensor enters the ready state. We assume that the discharge time (i.e., the time a sensor spends in the active state) and the recharge time (i.e., the time a sensor spends in the passive state) are random. Sensors in the ready state do not participate in sensing, and wait to get activated. Figure 1 explains the three different states, and the transitions between them. We assume that an external agent monitors the overall system state and decides on how many (and which ones) of the ready sensors should be activated at any moment. We address the decision problem faced by this external agent.

We make the following assumptions in our analysis.

Assumption 1 The discharge time and recharge time of any sensor are exponentially distributed with means \( 1/\mu_1 \) and \( 1/\mu_2 \) respectively. Moreover, \( \mu_1 \geq \mu_2 \).

Assumption 2 The energy level of a sensor does not change in the ready state.

In reality, the discharge and recharge times will depend on various random factors. Sensors can transmit information (resulting in energy usage) on the occurrence of “interesting” events, which
may be generated according to a random process. Similarly, the recharge time can depend on various random factors like intensity of sunlight etc. The exponential model of the discharge/recharge times is assumed for analytical tractability. Moreover, the optimal policies under this assumption depend only on the number of sensors in the different states in the system, and not their exact energy levels. Without Markovian properties, the system can be very difficult to analyze, and implementing the optimal decision policies (if they can be obtained) would require more detailed system information and additional overhead. The assumption $\mu_1 \geq \mu_2$ is based on the practical fact that the recharging process in batteries is typically a slow process, and can be expected to be much slower than the discharging process.

Assumption 2 basically states that a sensor remains in the fully charged state as long as it remains in the ready state. In reality, we would expect that energy will be drained even in the ready state, but probably at a fairly steady rate (possibly due to periodic polling). However, the energy discharge rate in the ready state can be expected to be much slower than the discharge rate in the active state, and is therefore not considered in our current analysis.

### 2.2 Problem Statement

We assume that the performance of the system is characterized by a continuous, non-decreasing, strictly concave function $U$. The utility of the system when there are $n$ active sensors is given by $U(n)$, and $U(0) = 0$.

Note that the strict concavity assumption merely states the fact that the system has diminishing returns with respect to the number of active sensors. We are interested in maximizing the time-average utility of the system. Let $n_P(t)$ denote the number of sensors in the active state at time $t$ under policy $P$. Define the time-average utility under policy $P$, $\bar{U}(P)$, as

$$\bar{U}(P) = \lim_{t \to \infty} \frac{1}{t} \int_0^t U(n_P(t)) \, dt.$$  \hspace{1cm} (1)

Then the our optimal decision problem is that of finding $P$ so that $\bar{U}(P)$ is maximized.

As an example of a practical utility function, consider the scenario where each sensor can detect an event with probability $p_d$. If the utility is defined as the probability that the sensing system is able to detect an event, then $\bar{U}(n) = 1 - (1 - p_d)^n$, where $n$ is the number of sensors that are active. Note that this utility function is strictly concave, and satisfies $U(0) = 0$.

As mentioned before, our decision problem is that of determining how many sensors to activate at any time, from the set of ready sensors. Note that if we activate more sensors, we gain utility in the short time-scale. However, if the number of active sensors is already large, since the utility function exhibits diminishing returns, we may want to keep some of the ready sensors “in store” for future use. In fact, as we see later, the structure of the optimal threshold policies justifies this intuition.

We assume that switching decisions can be taken at any instant of time. Clearly, these decisions would need to be taken only when the state of the overall system changes, i.e., the number of the sensors in the active, passive or ready states changes. In other words, these decisions need to be taken when some sensor makes a transition from the active to the passive state, or some sensor makes a transition from the passive to the ready state.

Practical constraints might require the sensor activation decisions to be executed in a distributed manner. Although our model assumes centralized decision-making, the analysis and results can be applied even when the decisions are taken in a decentralized manner. The implementation of our policies through distributed coordination amongst the sensors, and the related protocol issues, is outside the scope of this work.
2.3 Correlation Models

We consider two different correlation models of the discharge/recharge processes of the different sensors:

i) Independent Exponential (IE) Model: In this model, sensors are activated independently of one another, and all discharge and recharge times of all sensors are mutually independent.

ii) Batch Exponential (BE) Model: In this model, sensors are activated as a batch, and the discharge and recharge times of all sensors in a batch are identical. Therefore, the sensors in a batch always move through the system together. Since the discharge (recharge) times of all sensors in a batch is the same, we can define a single discharge (recharge) time for a batch. We assume that these batch discharge and recharge times satisfy Assumption 1, and all discharge and recharge times of all batches are mutually independent.

The two correlation models can be practically motivated in the following way. If the data transmission by a sensor (which is often the primary mode of energy expenditure) is independent of that of other active sensors, then the system is better represented by the IE model. However, in many scenarios, the sensors could perform data transmission collaboratively; in such a case, the BE model may be more appropriate. Note that these two models represent two extreme forms of correlation, and real-life situations can be expected to fall in between these two extremes.

Note that the optimal time-average utility (computed over all possible activation policies) could be different for the two correlation models. We denote optimal time-average utility for the IE and BE models as $\bar{U}^*_I$ and $\bar{U}^*_B$, respectively.

2.4 Threshold Activation Policies

Note that the set of all possible activation policies can be very large, and the structure of these policies can be very complex. Therefore, determining the optimal activation policy for the IE and BE models, can be very difficult, and evaluating the optimal time-average utilities, $\bar{U}^*_I$ and $\bar{U}^*_B$, can be computationally intensive. Therefore, we focus primarily on threshold activation policies.

A threshold activation policy with parameter $m$, is characterized as follows: a ready sensor $s$ is activated if the number of active sensors does not exceed $m$ after $s$ is activated; otherwise, $s$ is kept in the ready state. In other words, a threshold policy with parameter $m$ tries to maintain the number of active sensors as close to $m$ as possible. Note that with such a policy, the number of active sensors can never exceed $m$, and there cannot be any ready sensors in the system when the number of active sensors is less than $m$. The time-average utility for threshold activation policy with parameter $m$, are denoted by $\bar{U}_{T,I}(m)$ and $\bar{U}_{T,B}(m)$, for the IE and BE models, respectively.

3 Analysis

In this section, we compare the performance of threshold activation policies with respect the optimal activation policy. In the following, $\rho = \frac{\mu_1}{\mu_2} \geq 1$. For simplicity of exposition, we assume $\rho$ is an integer, and $N$ is divisible by $(p + 1)$, although our results can be generalized to the case where these assumptions do not hold.

3.1 Upper Bound on $\bar{U}^*_I$ and $\bar{U}^*_B$

Since the optimal time-average utility is difficult to compute, we obtain an upper bound on it, and compare the performance of threshold policies with this bound.
Theorem 1 The optimal time-average utility for the two correlation models, \( \bar{U}_I^* \) and \( \bar{U}_B^* \), are both upper-bounded by \( U\left(\frac{N}{1+\rho}\right) \), i.e.,
\[
\bar{U}_I^* \leq U\left(\frac{N}{1+\rho}\right) \quad \text{and} \quad \bar{U}_B^* \leq U\left(\frac{N}{1+\rho}\right).
\]

The proof of the above result involves concavity arguments and Jensen’s Inequality. Theorem 1 implies that the time-average utility under any policy can not be greater that \( U\left(\frac{N}{1+\rho}\right) \). Further, the bound is achieved exactly when all the sensors have deterministic discharge and recharge times of lengths \( 1/\mu_1 \) and \( 1/\mu_2 \), respectively. With random discharge/recharge times, the bound may not be tight; however, as we show below, it is fairly good bound in our case.

Now we derive worst-case bounds on the performance of threshold policies with respect to the optimal policy, for the two correlation models.

3.2 Threshold Activation Policies for the IE Model

Consider a threshold activation policy with parameter \( m \in \{1, 2, 3, ..., N\} \). Using steady state Markov chain analysis, the time-average utility of the system, \( \bar{U}_{T,I}(m) \), can be computed as
\[
\bar{U}_{T,I}(m) = \frac{\sum_{i=1}^{N} U(i) \alpha(i, m)}{\sum_{i=0}^{N} \alpha(i, m)}, \tag{2}
\]
where \( \alpha(i, m), i = 1, 2, ..., N, \) are defined as
\[
\alpha(i, m) = \begin{cases} \binom{N}{i} \rho^{-i} & \text{if } i \leq m, \\ \binom{N}{i} \frac{\rho^{-i}}{m^{m-i}} & \text{otherwise}. \end{cases} \tag{3}
\]

Then \( \bar{U}_{T,I}^* \), the optimal threshold-based time-average utility for the IE model, is defined as \( \bar{U}_{T,I}^* = \max_{m=1}^{N} \bar{U}_{T,I}(m) \).

Next we state a result for the threshold policy with parameter \( N \). Note that with a threshold of \( N \), once a sensor is completely recharged, it is immediately activated. In other words, no sensor is kept in the ready state.

Theorem 2 The time-average utility at threshold \( N \) for the IE model, \( \bar{U}_{T,I}(N) \), is lower-bounded by \( \frac{1}{2} U\left(\frac{N}{1+\rho}\right) \), i.e,
\[
\bar{U}_{T,I}(N) \geq \frac{1}{2} U\left(\frac{N}{1+\rho}\right).
\]

The proof of the above result involves concavity arguments and several approximations using the expression in (2). Theorem 2 implies that with a threshold of \( N \), the performance of the system will be within 50% of the optimal performance over all policies. Our numerical studies also show that the time-average utility of the system with a threshold of \( N \) is usually quite close to the optimal. However, the optimal threshold could in general be much less than \( N \). The best threshold policy can be found by finding the maximum of \( \bar{U}_{T,I}(m) \) over all \( m \in \{1, 2, 3, ..., N\} \), using the expression in (2). Theorem 2, in conjunction with Theorem 1, implies that the optimal threshold-based time-average utility, \( \bar{U}_{T,I}^* \), satisfies \( \bar{U}_{T,I}^* \geq \frac{1}{2} \bar{U}_I^* \).

It is possible obtain a stronger bound on the performance of the optimal threshold policy for the IE model. The derivation of this bound uses results from the analysis of the BE model, that we discuss next.
3.3 Threshold Activation Policies for the BE Model

Consider a threshold activation policy with parameter $m$. We assume that $N$ is a multiple of $m$. Initially, all $N$ sensors are fully charged, and $m$ of these are activated, and the remaining $N - m$ are in the ready state. It is easy to see that the sensors will become grouped into $c = N/m$ batches, each of size $m$, and always move through the different states in these batches. From our definition of a threshold activation policy, it follows that at most one batch can remain active at any time. Using steady-state Markov chain analysis, the time-average utility of the system, $\bar{U}_{T,B}(m)$, can be computed as

$$
\bar{U}_{T,B}(m) = U(m) \left(1 - \frac{\rho^c}{\sum_{i=0}^{c} \rho^i}\right).
$$

(4)

Let $S_N$ denote the set of all factors of $N$, i.e., all integers which divide $N$. Then $\bar{U}^*_{T,B}$, the optimal threshold-based time-average utility for the BE model, is defined as $\bar{U}^*_{T,B} = \max_{m \in S_N} \bar{U}_{T,B}(m)$.

Next we state an important bound on $\bar{U}^*_{T,B}$.

**Theorem 3** The optimal threshold-based time-average utility for the BE model, $\bar{U}^*_{T,B}$, is lower-bounded by $\frac{3}{4} U(N/(1+\rho))$, i.e.,

$$
\bar{U}^*_{T,B} \geq \frac{3}{4} U(N/(1+\rho))
$$

The proof of Theorem 3 involves concavity arguments, and certain approximations using the expression in (4). Theorem 3, together with Theorem 1, implies $\bar{U}^*_{T,B} \geq \frac{3}{4} \bar{U}^*_{B}$. Therefore, the performance of the best threshold policy is within a factor of $\frac{3}{4}$ of the optimal performance over all policies. The best threshold policy can be found by finding the maximum of $\bar{U}_{T,B}(m)$ over all $m \in S_N$, using the expression in (4). As we describe later, our numerical results show that this maximum is typically attained at some intermediate value of $m$. From the proof of Theorem 3, it can be also shown that a threshold of $N/(1+\rho)$ achieves the bound of $\frac{3}{4}$.

3.4 Comparison of IE and BE Models

The following result states that for each threshold $m$, the performance for the IE model is at least as good as that for the BE model.

**Theorem 4** For any $m \in S_N$, the time-average utility for the IE model, $\bar{U}_{T,I}(m)$, can be no less than the time-average utility under the BE model, $\bar{U}_{T,B}(m)$, i.e.,

$$
\bar{U}_{T,I}(m) \geq \bar{U}_{T,B}(m).
$$

The proof of Theorem 4 involves constructing equivalent queuing networks corresponding to the IE and BE models, and comparing the mean waiting times in these networks. In other words, the presence of correlation amongst the discharge and recharge times of sensors in a batch degrades system performance.

Theorems 3 and 4 allow us to improve our earlier bound on the performance of the optimal threshold policy for the IE model.

**Corollary 5** The optimal threshold-based time-average utility for the IE model, $\bar{U}^*_{T,I}$, is lower-bounded by $\frac{3}{4} U(N/(1+\rho))$, i.e.,

$$
\bar{U}^*_{T,B} \geq \frac{3}{4} U(N/(1+\rho))
$$

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Corollary 5, in conjunction with Theorem 1, implies $\bar{U}^*_I \geq \frac{3}{4} \bar{U}^*_T$. Therefore, for both the IE and BE models, the performance of the best threshold policy is within a factor of $\frac{3}{4}$ of the best achievable performance.

### 3.5 Availability-based Activation Policies

Next we describe a class of policies that are more general than the threshold policies. The analysis of this general class of policies for the BE model seems very difficult, and therefore we restrict our discussion to the IE model. An availability based activation policy is characterized by an activation function $f : \mathbb{Z}_+ \rightarrow \mathbb{Z}_+$ (where $\mathbb{Z}_+$ is the set of all non-negative integers), which satisfies the following assumption.

**Assumption 3** The activation function $f$ is non-decreasing and satisfies $f(i) \leq i$ for all $i \in \mathbb{Z}_+$. Moreover, $i - f(i)$ is non-decreasing for all $i \in \mathbb{Z}_+$.

Let us denote the set of all active and ready sensors as the set of available sensors. When the number of available sensors in the system is $i$, then an availability-based activation policy is characterized as follows: a ready sensor $s$ is activated if the number of active sensors does not exceed $f(i)$ after $s$ is activated; otherwise, $s$ is kept in the ready state. Note that this activation policy only depends on the number of available sensors in the system. The availability-based activation policy, as described above, always maintains $f(i)$ active sensors in the system when the number of available sensors in the system is $i$. Note that $i - f(i)$ represents the number of ready sensors, when the number of available sensors is $i$. Therefore, Assumption 3 implies that the number of ready sensors is a non-decreasing function of the number of available sensors. Note that a threshold policy with parameter $i$ falls within this class of policies, and the corresponding activation function in this case, denoted by $f_T$, is given by

$$f_T(i) = \begin{cases} i & \text{if } i \leq m, \\ m & \text{otherwise}, \end{cases} \quad (5)$$

where $m$ is the threshold.

The first step towards analyzing the performance of availability-based activation policies is to obtain the time-average utility of the system, $\bar{U}_A(f)$, as a function of the activation function $f$. Using Markov chain analysis, we obtain the following result.

$$\bar{U}_A(f) = \frac{\sum_{i=1}^{N} U(f(i)) \beta(i)}{\sum_{i=0}^{N} \beta(i)}, \quad (6)$$

where $\beta(i), i = 1, 2, ..., N$, are defined as

$$\beta(i) = \begin{cases} \frac{\rho^{-i}}{\rho^{-N}} \prod_{i'=i+1}^{N} f(i') & \text{if } i < N \\ \rho^{-N} & \text{if } i = N \end{cases} \quad (7)$$

Note that for a given activation function $f = (f(1), f(2), ..., f(N))$, we could use the above expression to obtain the time-average utility of the system. To obtain $f^*$, the activation function that maximizes $\bar{U}_A(f)$, we could evaluate the above expression at all possible points in the function space, and compare them to find the maximum. Utilizing the properties of the activation function, we obtain the following result.

**Lemma 6** The optimal activation function, $f^*$, can be obtained by at most $e^N$ evaluations of the function $\bar{U}_A(f)$. 

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Note that the above result implies that this enumeration-based method is not polynomial-time. Whether this enumeration can be done in polynomial time, for general utility functions, remains an open question. Due to the complexity of the approach, identifying the optimal policy in this manner can be practical only if \( N \) is not too large. For large \( N \), the optimal threshold policy is likely to be a better choice.

Our numerical results show that the performance of the optimal availability-based policy can be better than that of the optimal threshold policy, as we intuitively expect. Therefore, in general, the optimal threshold policy may not attain the best achievable performance. However, we observe that the difference between the performance of the optimal availability-based policy and the optimal threshold policy is typically small.

4 Numerical Results

In this section, we report results from numerical experiments on the performance of threshold policies for the IE and BE models under different parameter settings. For the utility function \( U(n) = 1 - (1 - p_d)^n \), we conduct numerical experiments for different values of \( p_d(= 0.1, 0.9) \),
Table 1: Ratio of optimal threshold-based time average utility and lower bound, for $p_d = 0.1$.

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(a) IE Model

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(b) BE Model

Table 2: Ratio of optimal threshold-based time average utility and lower bound, for $p_d = 0.9$.

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(a) IE Model

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(b) BE Model

To obtain the different values of $\rho$, we set $\mu_2 = 1$ and vary $\mu_1$. For each parameter setting, we compare the time-average utility of the system for different values of the threshold, using expressions (2) and (4). Figures 2 and 3 depict typical plots that describe the performance of the threshold policies in the presence of low (Figure 2) and high (Figure 3) probability of detection ($p_d$). Note that the figures show the time-average utilities $\bar{U}_{T,I}(m)$ and $\bar{U}_{T,B}(m)$ along with $U(N + \rho)$, the upper bound on the maximum achievable time-average utility. Figures 2 and 3 indicates that for both the BE and IE models the threshold value that maximizes the time average utility lies between 1 and $N$. Further, the optimal threshold is distinct from $\frac{N + \rho}{1 + \rho}$. For the BE model, when operating with a threshold greater than the optimal, the time-average utility decreases very rapidly with the threshold value. However, for the IE model the decrease is gradual and in many cases marginal. The rapid decrease in performance of the BE model for thresholds other than the optimal emphasizes the need to model and understand impact of correlation on system performance.

Tables 1 and 2 list the ratio of the time-average utility obtained at the optimal threshold ($\bar{U}_{T,I}^*$ or $\bar{U}_{T,B}^*$) to the lower bound of $\frac{3}{4}U(N + \rho)$. Note that this ratio must lie between 1 and $\frac{4}{3}$. A value close to 1 indicates a tight lower bound, whereas a value close to $\frac{4}{3}$ indicates that performance of the optimal threshold policy is close to the best achievable performance.

Table 1 indicates that for low values of the probability of detection $p_d$, the time average utility obtained by the optimal threshold policy for the BE model is very close to the lower bound. However, performance of the optimal threshold policy for the IE model is fairly close to the maximum value $U(N + \rho)$. Table 2 indicates that for high values of the probability of detection $p_d$, the time average utility obtained by the optimal threshold policy for both models is very close to the maximum value $U(N + \rho)$, although the performance for the IE model is slightly better, as expected. In summary, in most cases, the optimal threshold policies yield performance that are very close to that maximum achievable performance. When this is not the case (for instance, for the BE model and low values of $p_d$), the performance is fairly close to our lower bound of $\frac{3}{4}U(N + \rho)$. Further, the numerical experiments also indicate that our bounds are fairly robust to the choice of $N$ and $\rho$.

Finally, we provide an example which explicitly shows that there could exist policies that out-
perform the optimal threshold policy. We consider the class of availability-based activation policies, and find the optimal activation function \( f^* \) by the enumeration-based procedure described in Section 3.5. Figure 4 shows \( f^* \) for the parameters \( N = 16, \rho = 7 \) and \( p_d = 0.5 \). Note that \( f^* \) does not correspond to a threshold policy. The optimal threshold in this case is 2, and the corresponding activation function, \( f^*_T \), is also shown in the figure. However, \( f^* \) provides only about 0.52% improvement in performance over \( f^*_T \).

5 Future Directions

In this work, we have only considered the problem where all sensors have identical coverages. In a general sensor network, however, sensors will typically have different coverage areas. While our algorithms can be used locally in a large-scale network scenario as well, it is not clear how these approaches should be adapted so as to obtain provably small performance bounds in that case. This remains an important topic that merits further investigation.

Appendix I: Upper Bound on Time-Average Utility

Since the optimal time-average utility is difficult to compute, we obtain an upper bound on it, and compare the performance of threshold policies with this bound. In the following, \( \rho = \frac{\mu_1}{\mu_2} \geq 1 \). For simplicity of exposition, we assume \( \rho \) is an integer, and \( N \) is divisible by \((\rho + 1)\), although our results can be generalized to the case where these assumptions do not hold.

Proof of Theorem 1: The proof of the above result involves concavity arguments and Jensen’s Inequality [5]. Let \( f \) and \( p \) be measurable functions finite a.a. on \( \mathbb{R} \). Suppose, that \( fp \) and \( p \) are integrable on \( \mathbb{R} \), \( p \geq 0 \), and \( \int p 0 \). If \( \phi \) is convex in an interval containing the range of \( f \), then Jensen’s inequality states that:

\[
\phi \left( \frac{\int_R fp}{\int_R p} \right) \leq \frac{\int_R \phi(f)p}{\int_R p}
\]
Let $n(t)$ denote the number of sensors in the active state at time $t$. Since $U(.)$ is concave, substituting $\phi = U(.)$, $f = n(t)$ and $p = 1$ in the above, Jensen’s Inequality implies that:

$$U \left( \frac{\int_0^T n(t) dt}{T} \right) \geq \frac{\int_0^T U(n(t)) dt}{T}$$

Since, $U(.)$ is continuous, we have:

$$\lim_{T \to \infty} U \left( \frac{\int_0^T n(t) dt}{T} \right) \geq \lim_{T \to \infty} \frac{\int_0^T U(n(t)) dt}{T}$$

Define $\psi_i(t)$ such that $\psi_i(t) = 1$ if sensor $i$ is in active state at time $t$ and $\psi_i(t) = 0$ if sensor $i$ is in passive state at time $t$. Then, continuity of $U(.)$ also implies

$$\lim_{T \to \infty} U \left( \frac{\int_0^T n(t) dt}{T} \right) = U \left( \lim_{T \to \infty} \frac{\int_0^T \sum_{i=1}^N \psi_i(t) dt}{T} \right)$$

Since $\psi_i(t)$ is positive and bounded,

$$\lim_{T \to \infty} U \left( \frac{\int_0^T n(t) dt}{T} \right) = U \left( \lim_{T \to \infty} \frac{\int_0^T \sum_{i=1}^N \psi_i(t) dt}{T} \right)$$

Further, since all sensors are identical, for any $k$

$$U \left( \lim_{T \to \infty} \frac{\int_0^T n(t) dt}{T} \right) = U \left( N \lim_{T \to \infty} \frac{\int_0^T \psi_k(t) dt}{T} \right)$$

Since the times each sensor spends in active and passive states are independent, with mean $\frac{1}{\mu_1}$ and $\frac{1}{\mu_2}$, we have

$$\frac{1}{1 + \rho} \geq \lim_{T \to \infty} \frac{\int_0^T \psi_k(t) dt}{T}$$

where the equality holds if the sensor spends zero time in the ready state. Therefore we have

$$U \left( \frac{N}{1 + \rho} \right) \geq U \left( \lim_{T \to \infty} \frac{\int_0^T n(t) dt}{T} \right)$$

This implies

$$U \left( \frac{N}{1 + \rho} \right) \geq U \left( \lim_{T \to \infty} \frac{\int_0^T n(t) dt}{T} \right) \geq \lim_{T \to \infty} \frac{\int_0^T U(n(t)) dt}{T}$$
This implies that the time-average utility under any policy cannot be greater than $U(\frac{N}{1+\rho})$. In particular, the optimal time-average utility for the two correlation models, $\bar{U}_i^*$ and $\bar{U}_B^*$, are both upper-bounded by $U(\frac{N}{1+\rho})$, i.e.,

$$\bar{U}_i^* \leq U(\frac{N}{1+\rho}) \text{ and } \bar{U}_B^* \leq U(\frac{N}{1+\rho}).$$

### Appendix II: Performance of Threshold Policies for IE Model

For a system operating under a threshold policy with parameter $m$, at any given point in time there can be $k$ sensors in the active state, where $k = 1, \ldots, m$. The state of the system can be represented by the tuple $(i, j)$ where $i$ represents the number of active sensors and $j$ represents the number of sensors in the passive state. If $S$ represents the set containing the possible system states at any arbitrary point in time, then $S = \{(j, N - j) | j = 0, \ldots, m\} \cup \{(m, N - j) | j = m + 1, \ldots, N\}$. When the system is in state $(j, N - j)$, for $j = 0, \ldots, m$, $j$ sensors in the active state and the remaining $N - j$ sensors are in the passive state being charged. When the system is in state $(m, N - j)$, for $j = m + 1, \ldots, N$, $m$ sensors are active, $N - j$ sensors are passive, and $N - j - m$ sensors are in the ready state. Since charging and discharging times are assumed to be exponentially distributed, with mean $1/\mu_2$ and $1/\mu_1$ respectively, the time evolution of the system can be represented using a continuous time Markov chain (CTMC) defined on the state space $S$. The CTMC is positive recurrent and has a unique vector of steady state probabilities. Let $\rho = \frac{\mu_1}{\mu_2}$ and $p(i, j)$ denote the steady state probability of the system being in state $(i, j)$ for $(i, j) \in S$. Then $p(i, j)$ need to satisfy the following balance equations.

$$p(0, N)N\mu_2 = p(1, N - 1)\mu_1$$

$$p(i, N - i)((N - i)\mu_2 + i\mu_1) = p(i - 1, N - i + 1)(N - i + 1)\mu_2 + p(i + 1, N - i - 1)(i + 1)\mu_1 \quad \text{for } 1 \leq i \leq m - 1$$

$$p(m, N - m)((N - m)\mu_2 + m\mu_1) = p(m - 1, N - m - 1)(N - m + 1)\mu_2 + p(1, c - 2)m\mu_1$$

$$p(m, i)(i\mu_2 + m\mu_1) = p(m, i - 1)(i - 1)\mu_2 + p(m, i + 1)m\mu_1 \quad \text{for } 1 \leq i \leq N - m - 1$$

$$p(m, 1)\mu_2 = p(m, 0)m\mu_1$$

Using the above equations and the normalization equation $\sum_{(i, j) \in S} p(i, j) = 1$, we obtain the expressions for $p(i, j)$, $(i, j) \in S$.

$$p(i, N - i) = \binom{N}{i} \frac{p(0, N)}{\rho^i} \quad \text{for } 1 \leq i \leq m$$

$$p(m, i) = \binom{N}{i} \frac{p(0, N)}{\rho^i} \frac{i!}{m!m^{i-m}} \quad \text{for } m + 1 \leq i \leq N$$
where

\[ p(0, N) = \left[ \sum_{i=0}^{m} \binom{N}{i} \frac{1}{\rho^i} + \sum_{i=m+1}^{N} \binom{N}{i} \frac{1}{\rho^i (m!)(m-i)!} \right]^{-1} \]

The time average utility obtained using a threshold policy of \( m \) is given by:

\[ \bar{U}_{T,I} = \sum_{i=1}^{m} p(i, N-i) U(i) + \sum_{i=m+1}^{N} p(m, N-i) U(m) \]

**Proof of Theorem 2:** When the threshold \( m = N \), we have

\[ p(0, N) = (1 + 1/\rho)^N \]
\[ p(i, N-i) = \binom{N}{i} (1/\rho^i) \]

for \( 1 \leq i \leq N \)

The time average utility obtained using a threshold of \( m = N \) is:

\[ \bar{U}_{T,I}(N) = \sum_{i=0}^{N} p(i, N-i) U(i) \]
\[ = \sum_{i=0}^{N} U(i) \binom{N}{i} (1/\rho^i) \]

\[ = \sum_{i=1}^{w} U(i) \binom{N}{i} (1/\rho^i) + \sum_{i=w+1}^{N} U(i) \binom{N}{i} (1/\rho^i) \]

We define \( w = \frac{N}{\rho+1} \) and show that \( \frac{\bar{U}_{T,I}(N)}{\bar{U}_{T,I}(w)} \geq \frac{1}{2} \). Then:

\[ \bar{U}_{T,I}(N) \]
\[ = \sum_{i=0}^{N} \frac{U(i) \binom{N}{i} (1/\rho^i)}{(1+1/\rho)^N} \]
\[ = \sum_{i=1}^{w} \frac{U(i) \binom{N}{i} (1/\rho^i)}{(1+1/\rho)^N} + \sum_{i=w+1}^{N} \frac{U(i) \binom{N}{i} (1/\rho^i)}{(1+1/\rho)^N} \]

But from the concavity of \( U(.) \) we have, \( U(k) \geq (k/w)U(w) \) for \( k \leq w \) and \( U(k) \geq U(w) \) for \( k > w \). Hence

\[ \frac{\bar{U}_{T,I}(N)}{\bar{U}_{T,I}(w)} \geq \sum_{i=1}^{w} \frac{(k/w) \binom{N}{i} (1/\rho^i)}{(1+1/\rho)^N} + \sum_{i=w+1}^{N} \frac{\binom{N}{i} (1/\rho^i)}{(1+1/\rho)^N} \]
\[ \geq \frac{1}{2} \left( \frac{w}{w+1} \right) \left[ \sum_{i=1}^{w} \binom{N}{i} (1/\rho^i) \right] + \sum_{i=w+1}^{N} \binom{N}{i} (1/\rho^i) \]
\[ \geq \frac{1}{2} \left[ \sum_{i=1}^{N} \binom{N}{i} (1/\rho^i) \right] + \frac{w}{2} \left[ \sum_{i=1}^{w} \binom{N}{i} (1/\rho^i) \right] \]
\[ \geq \frac{1}{2} \]

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Therefore, for the independent exponential model, the following performance bound holds when the system operates with a threshold \( m = N \):

\[
\frac{1}{2} U \left( \frac{N}{1 + \rho} \right) \leq \bar{U}_{T,I}(N) \leq U \left( \frac{N}{1 + \rho} \right)
\]

Appendix III: Performance of Threshold Policies for BE Model

For a system operating under a threshold policy with parameter \( m \), at any given point in time there can be at most \( m \) sensors in the active state. Typically, the number of sensors is large enough so that the \( N \) sensors can be partitioned into \( c \) groups each consisting of \( m \) sensors, i.e. \( c = N/m \). Then the state of the system can be represented by the tuple \((i, j)\) where \( i \) represents the number of groups of sensors in the passive state. If \( S \) denotes the set of possible system states at any arbitrary point in time, then \( S = \{(0, c)\} \cup \{(1, c-j)\mid j = 1, \ldots, c\} \).

When the system is in state \((0, c)\), all \( N \) sensors are being charged and there are no sensors in the active state. When the system is in state \((1, c-j)\), for \( j = 1, \ldots, c \), \( m \) sensors are active, \( N-km \) sensors are passive, and \( N-(j+1)m \) sensors are in the ready state. Since charging and discharging times are assumed to be exponentially distributed with mean \( 1/\mu_2 \) and \( 1/\mu_1 \) respectively, the time evolution of the system can be represented using a continuous time Markov chain (CTMC) defined on the state space \( S \). We analyze this CTMC to characterize the performance of threshold policies.

Since the CTMC is positive recurrent, it has a unique vector of steady state probabilities. Let \( \rho = \frac{\mu_1}{\mu_2} \) and \( p(i, j) \) denote the steady state probability of the system being in state \((i, j)\) for \((i, j) \in S\). Then \( p(i, j) \) need to satisfy the following balance equations.

\[
\begin{align*}
p(0, c)cm_2 &= p(1, c-1)\mu_1 \\
p(1, k)[k\mu_2 + \mu_1] &= p(1, k+1)(k+1)\mu_2 + p(1, k-1)\mu_1 \quad \text{for } 1 \leq k \leq c-2 \\
p(1, c-1)[(c-1)\mu_2 + \mu_1] &= p(0, c)cm_2 + p(1, c-2)\mu_1 \\
p(1, 1)\mu_2 &= p(1, 0)\mu_1
\end{align*}
\]

Using the above equations and the normalization equation, \( \sum_{(i, j) \in S} p(i, j) = 1 \), we get:

\[
\begin{align*}
p(0, c) &= \frac{\rho^c}{\sum_{i=0}^{c} \rho^i i!} \\
p(1, k) &= \frac{\rho^k}{\sum_{i=0}^{c} \rho^i i!} \quad \text{for } 0 \leq k \leq c-1
\end{align*}
\]

The time average utility obtained using a threshold of \( m \) is given by:

\[
\bar{U}_{T,B}(m) = [1 - p(0, c)]U(m)
\]

**Proof of Theorem 3:** To prove the lower bound on \( \bar{U}_{T,B}(\hat{m}) \), it is sufficient to show that there exists an \( \hat{m} \), such that \( \bar{U}_{T,B}(\hat{m}) = \frac{3}{4} U \left( \frac{N}{1+\rho} \right) \). In particular, we show that the result holds for \( \hat{m} = \frac{N}{1+\rho} \), by considering two cases, namely \( \rho = 1 \) and \( \rho \geq 2 \).
Case 1 ($\rho = 1$)

Since $\rho = 1$, $\hat{m} = \frac{N}{1+\rho} = \frac{N}{2}$ and $c = \frac{N}{\hat{m}} = 2$. Hence,

$$\bar{U}_{T,B}(\hat{m}) = \left(1 - \frac{\rho^2}{2\pi} \sum_{i=0}^{\hat{m}} \frac{\rho^i}{i!}\right) U \left(\frac{N}{1+\rho}\right)$$

$$= \left(1 + \frac{\rho}{1 + \rho} + \frac{\rho^2}{2}\right) U \left(\frac{N}{1+\rho}\right)$$

$$= 0.8 U \left(\frac{N}{1+\rho}\right)$$

$$\geq 3 \frac{3}{4} U \left(\frac{N}{1+\rho}\right)$$

Case 2 ($\rho \geq 2$)

Since $\rho \geq 2$, $\hat{m} = \frac{N}{1+\rho} \leq \frac{N}{3}$ and $c = \frac{N}{\hat{m}} \geq 3$. Hence,

$$\bar{U}_{T,B}(\hat{m}) = \left(1 - \frac{c^c}{\sum_{i=0}^{c} \frac{c^i}{i!}}\right) U \left(\frac{N}{1+\rho}\right)$$

$$\geq \left(1 - \frac{\rho^c}{1 + \rho + \frac{\rho^2}{2}}\right) U \left(\frac{N}{1+\rho}\right)$$

$$\geq \left(1 - \frac{1}{4 + 2} \frac{1}{\rho^2}\right) U \left(\frac{N}{1+\rho}\right)$$

$$\geq \left(1 - \frac{1}{4}\right) U \left(\frac{N}{1+\rho}\right)$$

$$\geq 3 \frac{3}{4} U \left(\frac{N}{1+\rho}\right)$$

Therefore, for the batch exponential model, the following performance bound holds when the system operates under the optimal threshold $m^*$:

$$\frac{3}{4} U \left(\frac{N}{1+\rho}\right) \leq \bar{U}^*_T, B \leq U \left(\frac{N}{1+\rho}\right)$$

Appendix IV: Comparison of IE and BE Models

Proof of Theorem 4: We prove the above result by constructing equivalent queuing networks corresponding to the IE and BE models, and comparing the mean waiting times in these networks. Figure 5 depicts the equivalent queuing network representation of the IE model operating with a threshold of $m$. The $N$ sensors in the system correspond to customers in a closed queuing network with two stations, namely, station 1 and station 2. The charging time of the passive sensors are modelled as the service time of a sensor at a station 2. Station 2 has $N$ identical exponential servers
with service rate $\mu_2$, and sensors in the passive state correspond to sensors receiving service at one of these $N$ servers. Similarly, the discharging time of the active sensors are modelled as the service time at station 1. Station 1 has $m$ identical exponential servers with service rate $\mu_1$ and sensors in the active state correspond to sensors receiving service at one of these $m$ servers. Sensors waiting in queue for service at station 1 correspond to sensors in the ready state. It can be easily shown that the state space and the markov chain corresponding to the queuing network shown in Figure 5 is identical to that of the IE model described in section 5. Further it can be shown that maximizing the time average utility of corresponds to minimizing the average waiting time at station 1. Figure 5: Equivalent queuing network representation for the IE model

6 depicts the equivalent queuing network representation of the BE model operating with a threshold of $m$. Note that in this case the $N$ sensors in the system can be partitioned into $c$ batches of size $N/m$. The behavior of each batch of sensors correspond to that of $N/m$ customers in a closed queuing network with two stations, namely, station 1 and station 2. The charging time of the passive sensors are modelled as the service time of a sensor at a station 2. Station 2 has $N$ identical exponential servers with service rate $\mu_2$, and sensors in the passive state correspond to sensors receiving service at one of these $N$ servers. Similarly, the discharging time of the active sensors are modelled as the service time at station 1. Station 1 has a exponential server with service rate $\mu_1$ and sensors in the active state correspond to sensors receiving service at this station. Sensors waiting in queue for service at station 1 correspond to sensors in the ready state. It can be easily shown that the state space and the markov chain corresponding to the queuing network shown in Figure 6 is identical to that of the BE model described in section 5. Further it can be shown that maximizing the time average utility of corresponds to minimizing the average waiting time at station 1. Based on the equivalence described above, in order to show that the time average utility of a system operating under the independent exponential model is at least as large as the time average utility obtained when operating under the batch exponential model, it is sufficient to Figure 6: Equivalent queuing network representation for the BE model
show that the average waiting time at station 1 in the network shown in Figure is never greater than the average waiting time at station 1 in the network shown in Figure 6. To do so, we first construct an alternative queuing network representation of the BE model in Figure 7. Unlike the network in Figure 6, the network in Figure 7 is a multi-class network, with \( m \) classes of customers and \( c (= N/m) \) customers in each class. The discharging time of the active sensors is modelled as the service time at an exponential server with service rate \( \mu_1 \). It is easy to see that maximizing the time average utility of the system corresponds to minimizing the average waiting time for this class of customers at this server. Finally we compare queuing network shown in Figure 5 and 7 and note that these networks have the same population. Further, the characteristics of station 2 in both these networks are identical. Prior work by [4], [2] and the references therein, on comparing the effect of server pooling on network performance measures indicates that network in Figure 5 would have higher throughput than the network shown in Figure 7. Then Little’s law implies that the average residence time of sensors in the network shown in Figure 5 is less than that in network shown in Figure 7. This implies that, for a given value of \( \rho, N \), and threshold \( m \), the time average utility of a system operating under the independent exponential model is at least as large as the time average utility obtained when operating under the batch exponential model.

\[ \text{Figure 7: Alternative queuing network representation for the BE model} \]

**References**


