Also, we have the boundary conditions,

$$A]_{b=a} = 1 \quad \text{and} \quad B]_{b=a} = 0. \tag{15}$$

Hence, we have from (13)–(15)

$$A = \cosh\left[\sqrt{z_0 y_0}(b-a)\right], \quad B = (\sqrt{z_0/y_0})\sinh\left[\sqrt{z_0 y_0}(b-a)\right].$$

From Table I, we readily establish

$$C = (\sqrt{y_0/z_0})\sinh\left[\sqrt{z_0 y_0}(b-a)\right], \quad D = \cosh\left[\sqrt{z_0 y_0}(b-a)\right].$$

In conclusion, recognition of the interesting interrelationships among the chain parameters of a nonuniform transmission line should be helpful in the study of such lines.

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On the Self-Learning Scheme of Nagy and Shelton

***Abstract*—A theoretical pattern recognition model is constructed to explain an experimental self-corrective character recognition scheme which was recently described.**

Recently Nagy and Shelton[1] have described an experimental self-corrective character recognition scheme. In this letter a theoretical pattern recognition model is constructed to explain the essence of their scheme.

Consider the problem of classifying the letters "A" and "B". Each letter is scanned and converted into a pattern vector x of $m(\sim 96)$ features. Suppose a total of $N(\sim 5000)$ such vectors (mixed A's and B's) are given without classification. Two initial reference vectors $R_A(0)$ and $R_B(0)$ are computed according to

$$R_A(0) = \sum_{i \in A(0)} x(i)$$
$$R_B(0) = \sum_{i \in B(0)} x(i)$$

where the set $A(0)$ is an arbitrary subset of N pattern vectors which we initially designated as being the letter "A", and similarly for $B(0)$. R_A and R_B are assumed to have been normalized. A reasonable decision procedure to reclassify the vectors is to say

$$x(0) \in \text{"A"} \quad \text{if} \quad x^T(0)[R_A(0) - R_B(0)] > 0$$
$$x(0) \in \text{"B"} \quad \text{if} \quad x^T(0)[R_A(0) - R_B(0)] < 0.$$

Of course, the result of this classification using $[R_A(0) - R_B(0)]$ as the decision vector creates two new subsets $A(1)$ and $B(1)$. Let

$$\alpha(0) = R_A(0) - R_B(0) \tag{1}$$
$$\beta_i(0) = x^T(i)\alpha(0) \qquad i = 1 \cdots N. \tag{2}$$

Then we recompute a new decision vector

$$\alpha(1) = R_A(1) - R_B(1) = \sum_{i \in A(1)} \beta_i(0)x(i) + \sum_{i \in B(1)} \beta_i(0)x(i), \tag{3}$$

i.e., the $R_A(1)$ and $R_B(1)$ are weighted averages of the pattern vectors belonging to sets $A(1)$ and $B(1)$. Those vectors corresponding to larger $\beta_i(0)$ are weighted more since presumably these are more representative of their group. Equations (1)–(3) now constitute a recursive procedure for the self-determination of the decision vectors.

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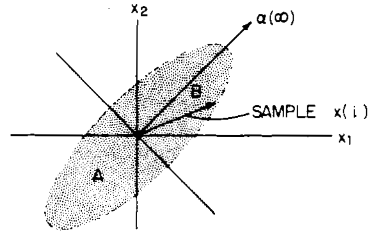

Fig. 1.

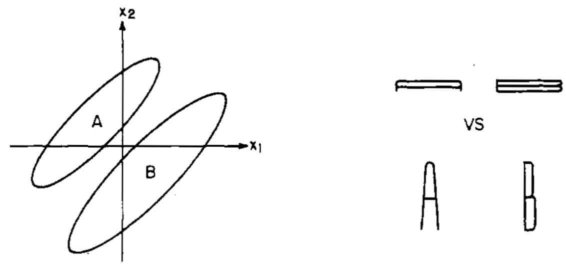

Fig. 2.

In matrix notation, (2) can be rewritten as

$$X^T\alpha(0) = \beta(0) \tag{2'}$$

where X is an $m \times N$ matrix with $x(i)$, $i = 1 \cdots N$ as its columns and $\beta(0)$ is an N vector of $\beta_i(0)$. Similarly, (3) now becomes in matrix notation

$$\alpha(1) = X\beta(0) = XX^T\alpha(0) \tag{3'}$$

or

$$\alpha(k+1) = XX^T\alpha(k). \tag{4}$$

Now it is a well-known fact in numerical analysis[2] that $\alpha(k)$ in (4) converges, for arbitrary $\alpha(0)$, to the eigenvector corresponding to the largest eigenvalue of the matrix XX^T which is the sample covariance matrix of the pattern vectors. In the case of $m=2$ the situation is shown in Fig. 1.

Thus, if "A" and "B" indeed have a distribution as portrayed in Fig. 1, as they most likely do in practical cases, then $\alpha(\infty)$ will be a good decision vector and the scheme of (4) represents a good self-teaching learning algorithm. The total of errors in classification will decrease per learning cycle on successive steps.

Note: 1) Equation (4) only preserves the directional properties of $\alpha(\infty)$. Thus, depending upon $\alpha(0)$, the "A" and "B" may be classified in reverse order.

2) If the distribution of "A" and "B" is as portrayed in Fig. 2, then the method will fail completely. In other words, this self-learning algorithm is really only a "dominant characteristic identification" method.

Actually, in the practical scheme of Nagy and Shelton, the elements of α, X, and β are only allowed binary values. Thus instead of (4), we have

$$\alpha(k+1) = \operatorname{sgn}[X \operatorname{sgn}(X^T\alpha(k))] \tag{5}$$

where we accept $+1$ and -1 as the two binary values. Convergence of (5) is, of course, different from (4). However, the authors believe that (4) conveys the essence of (5).

When there are more than two (52 or so) classes in practice in which we want to classify the patterns for this multiclass case with n classes, we may write the decision rule as

$$x^T(i)\left[R_A - \frac{1}{n-1}\sum_{j \neq A} R_j\right] \begin{cases} > 0 & x(i) \in A \\ < 0 & x(i) \notin A. \end{cases} \tag{6}$$

Thus for the class A we have a decision vector

$$\alpha_A = \left[R_A - \frac{1}{n-1}\sum_{j \neq A} R_j\right]. \tag{7}$$

This implies that we classify a pattern in class A by testing it against all the other classes. The above formulation now extends to this multiclass decision process very simply.

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Microwave Frequency Conversion Using Plasma Space-Charge Waves

***Abstract*—Presented is the observation of microwave frequency conversion due to parametric interaction between an incident extraordinary electromagnetic wave and a space-charge wave near the second cyclotron harmonic. The electrostatic mode is excited by the incident signal. It propagates across the magnetic field and is "quantized" because of boundary effects.**

This letter reports the observation of microwave frequency conversion due to parametric interaction of an incident microwave signal with space-charge waves propagating across a magnetoplasma column.

Bernstein [1] theoretically established the existence of electrostatic waves propagating within narrow frequency bands near the cyclotron harmonics. Waves of this type, which propagate across the magnetic field, do not suffer Landau damping and hence are easily excited. The dispersion relation for the low-temperature mode near the second cyclotron harmonic may be approximated by [2]

$$k_\perp^2 = \frac{[\omega^2 - \omega_p^2 - \omega_c^2][\omega^2 - 4\omega_c^2]}{3u_T^2\omega_p^2} \tag{1}$$

where $k_\perp$ = wave vector perpendicular to the magnetic field, ω = space-charge wave frequency, ω_p = local electron plasma frequency, ω_c = electron cyclotron frequency, and $u_T = (\mathrm{KT}/m)^{1/2}$ = electron thermal velocity.

In our experiment (Fig. 1), a plasma was generated by passing an electron beam through Ar gas at a pressure $\sim 10^{-3}$ torr. The plasma column was formed in a uniform magnetic field (0 to 4000 gauss). The primary electrons were provided by a hot cathode, and were accelerated by a grid placed just above the cathode. The plasma density was $\sim 10^{11}$–10^{12} electrons/cm^3. A microwave signal (frequency $\omega/2\pi = 4.6$ GHz, power ~ 10 mW) was coupled into the plasma by means of an antenna located outside the plasma column. The antenna was oriented with its plane perpendicular to the magnetic field, so that the electric field was polarized in the extraordinary mode. Only a small fraction of the microwave power could be coupled into the plasma. Another antenna at the opposite side coupled the converted signal into the 9.2 GHz receiver. A stopband filter in the transmitter circuit prevented its second harmonic signal from entering the receiver. The receiver was phase locked to the transmitter to suppress the background plasma noise.

The magnetic field was swept slowly in the vicinity of $\omega_c/2\pi = 2.3$ GHz so that the second cyclotron harmonic was very close to 4.6 GHz. We obtained resonances in the received signal at the second harmonic of the incident signal whenever the second cyclotron harmonic was a few hundred MHz above 4.6 GHz (Fig. 2).

It is known that an extraordinary wave may excite longitudinal electrostatic waves in the plasma, described by (1), by a mode conversion mechanism occurring when the upper hybrid condition, $\omega^2 = \omega_p^2(r) + \omega_c^2$, is satisfied [3] somewhere inside the plasma column. This is the case in our experiment. The extraordinary component of the incident signal is partially converted into an electrostatic mode, predicted by (1), having the same frequency (4.6 GHz). The electrostatic wave causes an electron density modulation and the corresponding modulation in the index of refraction of the medium is presented to the electromagnetic wave. The second harmonic of the incident signal is then parametrically generated by the interaction of the transverse incident wave with the electrostatic wave [4].

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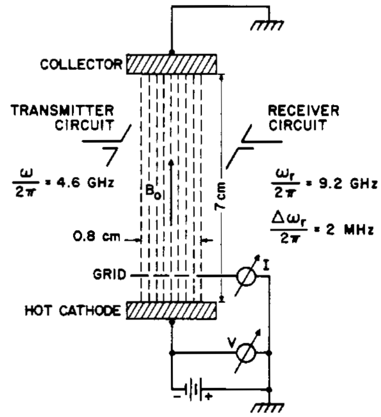

Fig. 1. The experimental system.

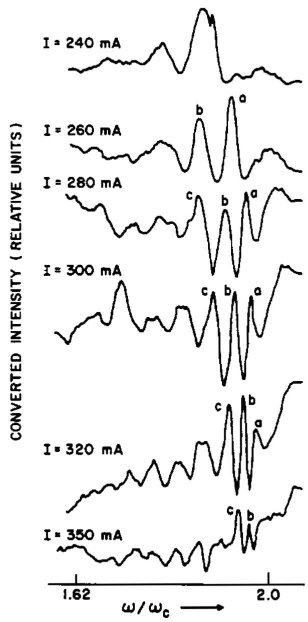

Fig. 2. Intensity of the converted signal at 9.2 GHz.

The fine structure on the received signal (Fig. 2) is caused by the quantization of $k_\perp$ because of the cylindrical boundary of the plasma column [2]. Each particular peak corresponds to a different eigenvalue of $k_\perp$. For each $k_\perp$, ω becomes a function of ω_p. We note that the frequency of the resonances occurs below the second cyclotron harmonic and moves towards the second harmonic as the current (or plasma density) is increased, in accordance with (1) and the experimental results of other investigators [2], [5].

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