2D Image Features

Two dimensional image features are interesting local structures. They include junctions of different types like 'Y', 'T', 'X', and 'L'. Much of the work on 2D features focuses on junction 'L', aks, corners.
Corner

Corners are the intersections of two edges of sufficiently different orientations.
Corner Detection

• Corners are important two dimensional features.

• They can concisely represent object shapes, therefore playing an important role in matching, pattern recognition, robotics, and mensuration.
Previous Research

- Corner detection from the underlying gray scale images.
- Corner detection from binary edge images (digital arcs).
Corner Detection from Gray Scale Image I

Corners are located in the region with large intensity variations.

Let $I_c$ and $I_r$ be image gradients in horizontal and vertical directions, we can defined a matrix $C$ as

- in two directions

\[
C = \begin{bmatrix}
\sum I_c^2 & \sum I_c I_r \\
\sum I_c I_r & \sum I_r^2
\end{bmatrix}
\]

where the sums are taken over a small neighborhood

Compute the eigenvalue of $C$, $\lambda_1$ and $\lambda_2$. If the minimum of the two eigen values is larger than a
threshold, the point is declared as a corner. It is good for detecting corners with orthogonal edges.
Corner Detection from Gray Scale Image II

Assume corners are formed by two edges of sufficiently different orientations, in a $N \times N$ neighborhood, compute the direction of each point and then construct a histogram of edgel orientations. A pixel point is declared as a corner if two distinctive peaks are observed in the histogram.
Corner Detection from Gray Scale Image II

Fit the image intensities of a small neighborhood with a local quadratic or cubic facet surface. Look for saddle points by calculating image Gaussian curvature (the product of two principle curvatures, see appendix A 5).
Corner Detection from Gray Scale Image II

Saddle points are points with zero gradient, and a max in one direction but a min in the other. Different methods are used to detect saddle points.
**Corner Detection from Digital Arcs**

- **Objective**

  Given a list of connected edge points (a digital arc) resulted from an edge detection, identify arc points that partition the digital arc into maximum arc subsequences.
Corner Detection from Digital Arcs

- Criteria
  - maximum curvature.
  - deflection angle.
  - maximum deviation.
  - total fitting errors.
A Statistical Approach

- Problem Statement

Given an arc sequence $S = \{(\hat{x}_n, \hat{y}_n) | n = 1, \ldots N\}$, statistically determine the arc points along $S$ that are most likely corner points.
Approach Overview

Input arc sequence S

Slide a window along S

Least-squares estimate $\theta_1$ and $\theta_2$

Estimate $\sigma_1^2$ and $\sigma_2^2$

Statistically test the difference between $\theta_1$ and $\theta_2$

P-value $\geq \alpha$?

Yes $\Rightarrow$ Not a corner

No $\Rightarrow$ A corner
Approach Overview (cont’d)

- Slide a window along the input arc sequence S.
- Estimate $\hat{\theta}_1$ and $\hat{\theta}_2$, the orientations of the two arc subsequences located to the right and left of the window center, via least-squares line fitting.
- Analytically compute $\sigma_1^2$ and $\sigma_2^2$, the variances of $\hat{\theta}_1$ and $\hat{\theta}_2$.
- Perform a hypothesis test to statistically test the difference between $\hat{\theta}_1$ and $\hat{\theta}_2$. 
Details of the Proposed Approach

- Noise and Corner Models
- Covariance Propagation
- Hypothesis Testing
Given an observed arc sequence \( S = \{ (\hat{x}_n \ \hat{y}_n)^t | n = 1, \ldots N \} \), it is assumed that \((\hat{x}_n, \hat{y}_n)\) result from random perturbations to the ideal points \((x_n, y_n)\), lying on a line \(x_n \cos \theta + y_n \sin \theta - \rho = 0\), through the following noise model:

\[
\begin{pmatrix}
\hat{x}_n \\
\hat{y}_n
\end{pmatrix} = \begin{pmatrix}
x_n \\
y_n
\end{pmatrix} + \xi_n \begin{pmatrix}
\cos \theta \\
\sin \theta
\end{pmatrix}; n = 1, \ldots, N
\]

where \(\xi_n\) are iid as \(N(0, \sigma^2)\).
Corner Model

\[ X \cos \hat{\theta}_1 + Y \sin \hat{\theta}_1 = \hat{\rho}_1 \]
\[ \Sigma_{\hat{\theta}_1, \hat{\rho}_1} \]
\[ \Sigma_{\hat{\theta}_2, \hat{\rho}_2} \]

\[
H_0 : \theta_{12} < \theta_0 \quad H_1 : \theta_{12} \geq \theta_0
\]

where \( \hat{\theta}_{12} = |\hat{\theta}_1 - \hat{\theta}_2| \), \( \theta_{12} \) is the population mean of \( \hat{\theta}_{12} \), and \( \theta_0 \) a threshold.
Covariance Propagation

• Problem statement

Analytically estimate $\Sigma_{\Delta \Theta}$, the covariance matrix of least-squares estimate $\hat{\Theta} = (\hat{\theta} \ \hat{\rho})^t$, in terms of the input covariance matrix $\Sigma_{\Delta X}$. 
Covariance Propagation (cont.)

From Haralick’s covariance propagation theory, define

\[ F(\hat{\Theta}, \hat{X}) = \sum_{n=1}^{N} (\hat{x}_n \cos \hat{\theta} + \hat{y}_n \sin \hat{\theta} - \hat{\rho})^2 \]

and

\[ g^{2 \times 1}(\Theta, X) = \frac{\partial F}{\partial \Theta} \]

then

\[ \sum_{\Delta \Theta} = \left( \frac{\partial g(X, \Theta)}{\partial \Theta} \right)^{-1} \left( \frac{\partial g(X, \Theta)}{\partial X} \right)^t \sum_{\Delta X} \left( \frac{\partial g(X, \Theta)}{\partial X} \right) \left[ \left( \frac{\partial g(X, \Theta)}{\partial \Theta} \right)^{-1} \right]^t \]
Define
\[ k = \begin{cases} 
+\sqrt{x^2 + y^2 - \rho^2} & \text{if } y \cos \theta \geq x \sin \theta \\
-\sqrt{x^2 + y^2 - \rho^2} & \text{otherwise}
\end{cases} \]
and
\[ \mu_k = \frac{1}{N} \sum_{n=1}^{N} k_n \]
\[ \sigma_k^2 = \sum_{n=1}^{N} (k_n - \mu_k)^2 \]
Geometric Interpretation of $k$
Covariance Propagation (cont.)

\[
\sum_{\Delta \Theta} = \sigma^2 \left( \begin{array}{cc}
\frac{1}{\sigma_k^2} & \frac{\mu_k}{\sigma_k^2} \\
\frac{\mu_k}{\sigma_k^2} & \frac{1}{N} + \frac{\mu_k^2}{\sigma_k^2} 
\end{array} \right)
\]
Hypothesis Testing

\[ H_0 : \theta_{12} < \theta_0 \quad H_1 : \theta_{12} \geq \theta_0 \]

where \( \theta_0 \) is an angular threshold and \( \theta_{12} \) is the population mean of RV \( \hat{\theta}_{12} = |\hat{\theta}_1 - \hat{\theta}_2| \).

Let

\[
T = \frac{\hat{\theta}_{12}^2}{\hat{\sigma}_{\theta_1}^2 + \hat{\sigma}_{\theta_2}^2}
\]

Under null hypothesis

\[
T \sim \chi_2^2
\]

if \( P(T) < \alpha \), then a corner else not a corner.
Corner Detection Example

Fig. 2. Extracted edges of building model for a RADUIS image with detected corners (represented by black square dots) overlaid.