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Antenna motion errors in bistatic SAR imagery

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Abstract
Antenna trajectory or motion errors are pervasive in synthetic aperture radar (SAR) imaging. Motion errors typically result in smearing and positioning errors in SAR images. Understanding the relationship between the trajectory errors and position errors in reconstructed images is essential in forming focused SAR images. Existing studies on the effect of antenna motion errors are limited to certain geometries, trajectory error models or monostatic SAR configuration. In this paper, we present an analysis of position errors in bistatic SAR imagery due to antenna motion errors. Bistatic SAR imagery is becoming increasingly important in the context of passive imaging and multi-sensor imaging. Our analysis provides an explicit quantitative relationship between the trajectory errors and the positioning errors in bistatic SAR images. The analysis is applicable to arbitrary trajectory errors and arbitrary imaging geometries including wide apertures and large scenes. We present extensive numerical simulations to validate the analysis and to illustrate the results in commonly used bistatic configurations and certain trajectory error models.

Keywords: synthetic aperture radar, bistatic radar, antenna motion errors, autofocus

1. Introduction

Synthetic aperture radar (SAR) imaging critically depends on the accuracy of antenna positions. However, antenna trajectory errors, which are also referred to as motion measurement or platform motion errors, are often unavoidable in SAR imaging. The motion errors are fundamentally due to the limitations of motion measurement systems. The lack of accurate
trajectory information results in geometric degradation in images where scatterers may be mispositioned or smeared. Understanding the effects of antenna trajectory errors on the reconstructed images is essential in developing techniques to correct for such degradations. In this paper, we present a quantitative analysis of positioning errors in bistatic SAR imagery due to antenna trajectory errors.

The antenna trajectory errors and the resulting geometric distortions in reconstructed images have been studied before [1–9]. However, all of these studies [1–9] are limited to certain configurations and trajectory error models and rely on certain simplifying assumptions. [1–4, 6–8] consider monostatic configuration and straight linear trajectory. [5] considers a bistatic configuration with straight linear transmitting and receiving antenna trajectories under far-field and small scene assumptions. Additionally, all of these studies describe the positioning errors semi-quantitatively where the positioning error is analyzed only in certain directions or described in qualitative terms. [9] describes target positioning errors for a bistatic configuration. However, this study is limited to a certain geometry where the transmitter is satellite-based and the two ground-based receivers are stationary. The study analyzes the positioning errors due to measurement errors in the satellite orbit and receiver locations.

The analysis and modeling of SAR motion errors and autofocus are interrelated, as any autofocus method requires an understanding of motion errors. In the past three decades, several SAR autofocus methods have been developed [1, 5, 10–27]. From the perspective of the current study, these methods can be classified into two categories. The first category of autofocus techniques rely on explicit phase error models [1, 5, 10–16]. These models vary from parametric polynomial models to those that are extracted from image degradations. The second category of autofocus techniques is based on optimization of certain image quality metrics, such as sharpness, contrast, entropy etc. While some of these image quality optimization methods are applied to reconstructed SAR images [17–22], others formulate SAR image formation and phase error estimation as a joint optimization problem [23–27].

In this paper, we present a theory to analyze positioning errors in bistatic SAR imagery due to antenna trajectory errors. Our analysis and results are applicable to both monostatic and bistatic configurations and arbitrary antenna trajectories [28–31], non-flat ground topography, as well as arbitrary antenna motion error models. The results can be easily extended to multi-static SAR imaging and SAR imaging exploiting multiple scattering [32, 33]. Our analysis does not rely on simplifying assumptions, such as small scene, short aperture or linear wavefront curvatures. We provide an explicit, analytic relationship between the three-dimensional antenna trajectory errors and two-dimensional target positioning errors. Our analysis can predict antenna motion errors which include space varying displacement and smearing that can be predicted given the antenna errors and imaging geometry. In addition to the general positioning error analysis, we consider specific trajectory error models, including low- and high-frequency trajectory errors and provide corresponding error analysis. Existing results on antenna motion errors can be derived as a special case of our analysis.

We perform our analysis in the backprojection (BP) based image reconstruction framework. BP-based reconstruction methods avoid many of the limiting assumptions required by fast fourier transform based methods, such as range-Doppler or polar format algorithms [1, 34, 35]. They can also be implemented efficiently by using fast BP algorithms [36] or fast fourier integral operator computation methods [37], and by utilizing parallel processing on graphics processing units [38]. Our approach is based on microlocal analysis [39–41]. As a result, the underlying methodology is applicable to analyze target positioning errors due to other error sources, such as variations in wave-speed, as well as to analyze positioning errors in other SAR modalities [42–46]. We present extensive numerical simulations to demonstrate the theoretical results for different geometries, configurations and trajectory error models. In
we present the antenna motion error analysis for the conventional monostatic SAR traversing a straight linear trajectory.

While our primary interest lies in radar imaging, the analysis introduced in this paper is also applicable to synthetic aperture imaging problems in acoustics, geophysics or microwave imaging.

The rest of the paper is organized as follows. In section 2, we present an overview of the filtered BP (FBP) image formation method. In section 3, we develop a theory to analyze the positioning errors in reconstructed SAR images due to antenna trajectory errors and provide explicit algebraic relationships. In section 4, we describe antenna trajectory error models. In section 5, we consider specific error models and reduce the general relationships to specific ones. Section 6 presents numerical simulations. Section 7 concludes the paper.

We use the following notational conventions throughout the paper. The bold Roman, bold italic and Roman lower-case letters are used to denote variables in \( \mathbb{R}^3 \), \( \mathbb{R}^2 \) and \( \mathbb{R} \), respectively, i.e., \( z = (x, z) \in \mathbb{R}^3 \), with \( x \in \mathbb{R}^2 \) and \( z \in \mathbb{R} \). The calligraphic letters (\( \mathcal{F}, \mathcal{K} \) etc.) are used to denote operators.

## 2. FBP image formation

In [48], we developed a FBP method for bistatic SAR image formation applicable to arbitrary flight trajectories and non-flat topography. In this section, we briefly describe the BP-based image formation.

Let \( x = [x, \psi(x)] \in \mathbb{R}^3 \) denote the Earth’s surface, where \( x = [x_1, x_2] \in \mathbb{R}^2 \) and \( \psi : \mathbb{R}^2 \to \mathbb{R} \) is a known function for the ground topography. Furthermore, we assume that the scattering takes place in a thin region near the surface. For a pair of transmitting and receiving antennas traversing the trajectories \( \gamma_T(s) \) and \( \gamma_R(s) \), \( s \in [s_0, s_1] \subseteq \mathbb{R} \), the received signal \( d \) can be modeled as follows [48]:

\[
d(s, t) \approx F[T](s, t),
\]

\[
d(s, t) = \int e^{-i\omega(t-R(s,x)/c_0)}A(x, s, \omega)T(x)\,d\omega\,dx,
\]

where \( t \) denotes the fast-time, \( s \) denotes the slow-time, \( c_0 \) denotes the speed of light in free space, and \( \omega \) denotes the temporal frequency. \( R \) is the bistatic range given by

\[
R(s, x) = |\gamma_T(s) - x| + |x - \gamma_R(s)|.
\]

\[ T(x) \] denotes the surface reflectivity and \( A \) is a complex amplitude function that includes the transmitter and receiver antenna beampatterns, the transmitted waveform and geometrical spreading factors. We refer to \( F \) as the forward map. We assume that \( A \) is a slow-varying function of \( \omega \). Under this and other mild assumptions the forward map \( F \) becomes a fourier integral operator (FIO) [49, 50].

Microlocal analysis is a mathematical theory of FIOs, high frequency analysis and singularities. In an image, singularities can be thought of as point-like and curvilinear structures or simply edges. Microlocal analysis provides a framework to design approximate inversion methods for FIOs. These methods are edge-preserving in the sense that a visible edge\(^1 \) in the scene is reconstructed at the correct location and orientation in the reconstructed image. In the context of synthetic aperture imaging, this means that a target is reconstructed at its correct location. Microlocal analysis provides concepts and tools to analyze where an edge in the

\(^{1}\) The term ‘visible edges’ refers to the edges of a scene reflectivity whose information is acquired by the imaging system given the imaging geometry.
scene is propagated in the received signal and, from the received signal, to the reconstructed image by the underlying FIOs [49–51].

We form the image of the scene, \( \tilde{T} \), by another FIO \( \mathcal{K} \) in the form of a FBP operator as follows:

\[
\tilde{T}(z) = \mathcal{K}[d](z) = \int e^{i\omega(t-R(x,z)/c)}Q(z, s, \omega)d(s, t)dsdt,
\]

where the filter \( Q \) can be chosen according to a variety of criteria [52, 53]. We refer to \( \mathcal{K} \) as the inverse map. Note that \( z = [z, \psi(z)] \in \mathbb{R}^3 \) denotes the position variable in the reconstructed image.

Substituting (1) into (3), we obtain the point spread function associated with the image fidelity operator \( \mathcal{K}\mathcal{P} \) as follows:

\[
L(z, x) = \int e^{i\phi(a,x,x)}Q(z, s, \omega)A(x, s, \omega)dsdt,
\]

where \( \phi(a, z, x, s) = \omega[R(s, x) - R(s, z)] \).

Hörmander–Sato lemma [41, 49–51, 54] tells us that the BP operator reconstructs an edge at location \( x \) in the scene at position \( z \) in the image provided that \( x \) and \( z \) satisfy the following criticality conditions:

\[
\partial_t^\omega \phi(a, z, x, s) = 0 \Rightarrow R(s, z) = R(s, x),
\]

\[
\partial_s^\omega \phi(a, z, x, s) = 0 \Rightarrow f(s, z) = f(s, x),
\]

where \( R \) is the bistatic range defined in (2) and \( f \) is the bistatic Doppler frequency defined as

\[
f(s, x) = \omega \left[ T_T(s) \cdot (R_T(s) - x) + R_R(s) \cdot (R_R(s) - x) \right].
\]

(5) and (6) show that a target reconstructed at positions \( z \) in the image has the same bistatic range and bistatic Doppler as that of a target at position \( x \) in the scene at slow-time \( s \). For a target to be reconstructed at the correct position, the set of points \( x, z \) satisfying (5) and (6) must include \( x = z \). All other solutions of (5) and (6) can be regarded as artifacts some of which can be avoided by proper choice of illumination pattern and antenna trajectories, such as the well-known right-left ambiguity [55] artifact.

### 3. Analysis of positioning errors in bistatic SAR images due to antenna trajectory errors

In this section we analyze the target positioning errors in SAR images that are reconstructed with erroneous antenna trajectories. The analysis provides quantitative relationship between the target positioning errors and the antenna trajectory errors.

Let \( \gamma_T(s) \) and \( \gamma_R(s) \) represent the ideal trajectories of the transmitting and receiving antennas, respectively and \( \hat{\gamma}_T(s) \) and \( \hat{\gamma}_R(s) \) represent the corresponding true trajectories. The ideal trajectories may correspond to predesigned trajectories, such as the straight linear flight path or they may be measured trajectories that deviate from the true trajectories due to the limitations of the underlying measurement system. We assume that ideal trajectories are used in the imaging process, as such they are erroneous.
Thus, we write
\[ γ_T(s) = γ'_T(s) + e_1 Δγ_T(s), \]
\[ γ_R(s) = γ'_R(s) + e_2 Δγ_R(s), \]
where \( e_1 Δγ_T(s) \) and \( e_2 Δγ_R(s) \) denote the errors in the transmitting and receiving trajectories, respectively. \( e_1, e_2 \in \mathbb{R} \) are small constants.

Note that SAR data depend on true antenna trajectories, while image reconstruction uses ideal or presumed antenna trajectories. Since BP reconstructs SAR images on the basis of matching the range and spatial Doppler of scatterers with those of reconstructed singularities, as shown by (5) and (6), the range and Doppler of the scatterers are mismatched when (8) and (9) are used in the image formation. As a result, the scatterer at position \( x = z \) in the scene is mapped to an erroneous position \( z_e = z + Δz \) in the image. Thus, we have
\[ R(γ'_T(s), γ'_R(s), z + Δz) - R(γ_T(s), γ_R(s), x) = 0, \]
\[ f(γ'_T(s), γ'_R(s), z + Δz) - f(γ_T(s), γ_R(s), x) = 0. \]

Where \( Δz \) denotes the target positioning error due to trajectory errors \( e_1 Δγ_T(s) \) and \( e_2 Δγ_R(s) \). Note that since \( x \) is a location on the ground, \( Δz = [Δz, Δz_3] \) where \( Δz_3 = ψ(Δz + ψ(Δz)) - ψ(Δz) \).

Note that in (10) and (11) we redefine
\[ R(γ_T(s), γ_R(s), x) = |γ_T(s) - x| + |x - γ_R(s)|, \]
\[ f(γ_T(s), γ_R(s), x) = o[γ_T(s) · (γ_T(s) - x) + γ_R(s) · (γ_R(s) - x)]. \]

Our objective is to determine the first order approximation to the position error \( Δz \) due to the trajectory errors. In order to determine \( Δz \), we use (8) and (9) to express (10) and (11) in terms of true antenna trajectories. Next, we assume that \( e_1, e_2 \to 0 \) are small and expand (10) and (11) in Taylor series around \( e_1 = 0 \) and \( e_2 = 0 \) and keep the first-order terms in \( e_1 \) and \( e_2 \). We then obtain
\[ -e_1 \partial_s R(γ_T(s), γ_R(s), z) \bigg|_{s=0} - e_2 \partial_s R(γ_T(s), γ_R(s), z) \bigg|_{s=0} \]
\[ + \partial_z R(γ_T(s), γ_R(s), z) \cdot Δz = 0, \]
\[ -e_1 \partial_s f(γ_T(s), γ_R(s), z) \bigg|_{s=0} - e_2 \partial_s f(γ_T(s), γ_R(s), z) \bigg|_{s=0} \]
\[ + \partial_z f(γ_T(s), γ_R(s), z) \cdot Δz = 0. \]
\[(14)\] simplifies to
\[
\begin{align*}
\Delta z \cdot \Xi(s, z) &= \frac{1}{|\Xi(s, z)|} \left[ e_1 \Delta \gamma_T(s) \cdot \left( \gamma_T(s) - z \right) + e_2 \Delta \gamma_R(s) \cdot \left( \gamma_R(s) - z \right) \right]
\end{align*}
\]
(16)
where
\[
\Xi(s, z) = D\psi(z) \left[ \left( \gamma_T(s) - z \right) + \left( \gamma_R(s) - z \right) \right].
\]
(17)
with
\[
D\psi(z) = \begin{bmatrix}
1 & 0 \\
0 & 1 \\
\partial \psi/\partial z_1 \\
\partial \psi/\partial z_2
\end{bmatrix}
\]
performing the projection onto the tangent ground plane. Note that \(\hat{u}\) denotes the unit vector in the direction of \(u\), e.g. \(\Xi(s, z)\) is the unit vector in the direction of \(\Xi(s, z)\).

Let \(\mathbf{b} = \gamma_T(s) - z + \gamma_R(s) - z\).
(19)
We refer to the unit vector of \(\mathbf{b}\), \(\hat{\mathbf{b}}\) as the look-direction of the bistatic SAR. Note that \(\hat{\Xi}\) is the projection of the three-dimensional bistatic look-direction onto the tangent plane of the ground topography. For the rest of the paper, we use the term "bistatic look-direction" to refer to \(\hat{\mathbf{b}}\) and \(\hat{\Xi}\) unless the distinction is unclear from the context.

Similarly, \((15)\) simplifies to
\[
\begin{align*}
\left[ \Delta z \cdot \Xi(s, z) \right] &= \frac{1}{|\Xi(s, z)|} \left[ e_1 \Delta \gamma_T^1(s) \cdot \left( \gamma_T(s) - z \right) + \gamma_T(s) \cdot \left( \gamma_T(s) - z \right) \right] \\
&\quad + e_2 \Delta \gamma_R^1(s) \cdot \left( \gamma_R(s) - z \right) + \gamma_R(s) \cdot \left( \gamma_R(s) - z \right)
\end{align*}
\]
(20)
where \(\Xi(s, z)\) is the derivative of \(\Xi(s, z)\) with respect to \(s\) and \(\hat{\Xi}(s, z)\) denotes the unit vector in the direction of \(\hat{\Xi}(s, z)\). This vector is given by
\[
\hat{\Xi}(s, z) = D\psi(z) \begin{bmatrix}
\gamma_T^1(s) \\
\gamma_R^1(s)
\end{bmatrix}
\]
(21)
In \((20)\) and \((21)\), \(\Delta \gamma_T(s)\) and \(\Delta \gamma_R(s)\) denote the components of the transmitter/receiver trajectory errors and transmitter/receiver velocities perpendicular to the corresponding look-directions, \((\gamma_T(s) - z) = \gamma_T^1(s)\). They are given explicitly as follows:
\[
\begin{align*}
\Delta \gamma_T^1(s) &= \Delta \gamma_T(s) - \gamma_T(s) \cdot \left( \gamma_T^1(s) - z \right) \\
\Delta \gamma_R^1(s) &= \Delta \gamma_R(s) - \gamma_R(s) \cdot \left( \gamma_R^1(s) - z \right)
\end{align*}
\]
(22)
\[
\begin{align*}
\gamma_T^1(s) &= \gamma_T(s) - \gamma_T(s) \cdot \left( \gamma_T^1(s) - z \right) \\
\gamma_R^1(s) &= \gamma_R(s) - \gamma_R(s) \cdot \left( \gamma_R^1(s) - z \right)
\end{align*}
\]
(23)
Let
\[
\mathbf{b}_\perp = \frac{\dot{\gamma}_T(s)}{|\gamma_T(s) - z|} + \frac{\dot{\gamma}_R(s)}{|\gamma_R(s) - z|}.
\] (24)

Since the direction of \(\mathbf{b}_\perp\) depends on the components of the antenna velocities perpendicular to the antenna look-directions, \((\gamma_T(s) - z)\), we refer to \(\mathbf{b}_\perp\) as the \textit{transverse bistatic look-direction} in contrast with the definition of bistatic look-direction given in (19). \(\hat{\Xi}\) is the projection of the three-dimensional transverse bistatic look-direction onto the tangent ground plane. For the rest of the paper, we use the term \textit{bistatic transverse look-direction} to refer to \(\mathbf{b}_\perp\) and \(\hat{\Xi}\). Note that this direction depends on the transmitting and receiving antenna velocities. Furthermore, \(\hat{\Xi}\) and \(\hat{\Xi}'\) are not necessarily orthogonal. For the derivation of (16) and (20), see appendices A and B.

**Definitions**: For ease of exposition, for the rest of the paper, we define the following terms and notation.

- Target positioning errors -
  - Radial positioning error : \(\Delta z^\Xi = \Delta z \cdot \hat{\Xi}(s, z)\).
  - Transverse positioning error : \(\Delta \xi^\Xi = \Delta z \cdot \hat{\Xi}(s, z)\).

- Antenna trajectory and velocity errors -
  - Radial Tx/Rx (transmitter/receiver) trajectory error : \(\Delta \gamma_{T,R}^r(s) = \Delta \gamma_{T,R}(s) \cdot (\gamma_{T,R}(s) - z)\).
  - Radial Tx/Rx velocity error : \(\Delta \dot{\gamma}_{T,R}^r(s) = \Delta \dot{\gamma}_{T,R}(s) \cdot (\gamma_{T,R}(s) - z)\).

  \(\Delta \gamma_{T,R}^r(s) = \Delta \gamma_{T,R}(s) - \left[ \Delta \gamma_{T,R}(s) \cdot (\gamma_{T,R}(s) - z) \right]\).

**Remarks**: We now summarize the implications of (16) and (20).

- The positioning error \(\Delta z\) for a target located at \(z\) is given by the solution of (16) and (20).
- (16) gives the target positioning error along the bistatic look-direction and (20) describes the positioning error in the transverse bistatic look-direction.
- From (16) and (20), we see that the radial position error \(\Delta z^\Xi\) mainly depends on the radial antenna trajectory errors, \(\Delta \gamma_{T,R}^r\). The transverse position error \(\Delta \xi^\Xi\) mainly depends on the radial antenna velocity errors, \(\Delta \dot{\gamma}_{T,R}^r(s)\), the antenna velocities, \(\dot{\gamma}_{T,R}\), the transverse antenna trajectory errors, \(\Delta \gamma_{T,R}^r\), and the antenna-to-target range, \(|\gamma_{T,R}(s) - z|\). Note that for the transverse position error \(\Delta \xi^\Xi\), the radial antenna velocity errors contribute much more than other terms due to the large range terms in the denominators of the second and fourth terms of (20). Thus, the radial trajectory error not only determines the radial position error \(\Delta z^\Xi\) as expected, but also makes a significant
contribution to the transverse position error $\triangle \xi_z$. This is consistent with the fact that wideband SAR imaging relies on range measurements along the antenna look-directions.

- The contributions of the transverse antenna trajectory errors, $\triangle Y^T_z(s)$ and $\triangle Y^R_z(s)$, to the transverse target positioning error $\triangle \xi_z$, can be significant since these trajectory errors are scaled by the range of the transmitting/receiving antennas. Depending on the geometric configuration, the transmitting and receiving antenna trajectory errors may have different contributions to target positioning errors. For example, in bistatic scenarios where a satellite-borne SAR is used as an illuminator of opportunity and where the receiver is airborne [56, 57] or mounted on a hill [58], the transverse trajectory error of the receiving antenna, $\triangle Y^R_z(s)$, may contribute more to transverse position error, $\triangle \xi_z$, than that of the transmitting antenna, $\triangle Y^T_z(s)$.

- The $z$ dependency of (16) and (20) accounts for the space-varying nature of the antenna trajectory errors on the reconstructed images.

- The $s$ dependency of the position errors in (16) and (20) explains the smearing observed in the SAR images. Thus, the space-varying positioning errors and smearing due to the antenna trajectory errors observed in the reconstructed images can be analyzed quantitatively using (16) and (20).

- Let $\mathbf{\hat{z}} = [\hat{z}_1, \hat{z}_2], \mathbf{\hat{z}} = [\hat{z}_1, \hat{z}_2]$ and $\Delta \mathbf{z} = [\Delta z_1, \Delta z_2]$. We can express the positioning errors given in bistatic and transverse bistatic look-directions in geolocation coordinates by jointly solving (16) and (20):

$$\Delta z_1 = \frac{\mathbf{\hat{z}}_1 \Delta z^T - \mathbf{\hat{z}}_1 \Delta z^R}{|\mathbf{\hat{z}}_1 \mathbf{\hat{z}}_2 - \mathbf{\hat{z}}_2 \mathbf{\hat{z}}_1|}, \quad (30)$$

$$\Delta z_2 = \frac{\mathbf{\hat{z}}_1 \Delta z^T - \mathbf{\hat{z}}_1 \Delta z^R}{|\mathbf{\hat{z}}_1 \mathbf{\hat{z}}_2 - \mathbf{\hat{z}}_2 \mathbf{\hat{z}}_1|}. \quad (31)$$

For ease of exposition, we refer to $\Delta z_1$ and $\Delta z_2$ as the horizontal and vertical positioning errors, respectively. In special geometries, such as a monostatic boresight SAR flying along the $z_1$ axis, the horizontal and vertical positioning errors in the reconstructed image are consistent with the radial and transverse positioning errors.

4. Antenna trajectory error models

All classic SAR image formation algorithms with the exception of the BP-based methods are designed for straight linear trajectories. Therefore any deviation from the straight linear trajectory is regarded as erroneous. This deviation from the ideal or pre-designed trajectory can be classified into two classes depending on its variation over the aperture [2]: low-frequency errors, which have periods larger than the coherent integration time of the whole aperture; and high-frequency errors, which vary rapidly over the aperture and have periods less than the aperture integration time. It has been shown that the low-frequency trajectory errors typically originate from the measurement errors in the velocity, acceleration and even acceleration rate of the antennas, which are usually modeled by low-order polynomials (up to the third order [1, 2]). The high-frequency trajectory errors are induced by the uncompensated fluctuations in antenna phase center and random measurement errors [1]. This type of trajectory errors are modeled by sinusoidal functions and noise-like wideband random signals, respectively.
Let $\Delta \gamma_{LR}(s)$ denote low-frequency trajectory error of the transmitting/receiving antenna. We assume

$$\Delta \gamma_{LR}(s) \approx \Delta \gamma(s) + \Delta \gamma_{v}(s) + \frac{\Delta \gamma_{a}(s)}{2},$$

(32)

where $\Delta \gamma_{0,R}(T)$ represents the initial transmitting/receiving antenna position error, $\Delta \gamma_{v}(s)$ and $\Delta \gamma_{a}(s)$ represent the transmitting/receiving antenna velocity and antenna acceleration errors. For ease of exposition, we refer to $\Delta \gamma_{0,R}(T)$, $\Delta \gamma_{v}(s)$ and $\Delta \gamma_{a}(s)$ as the constant, linear and quadratic trajectory errors, respectively.

The high-frequency sinusoidal trajectory error is modeled by

$$\Delta \gamma_{H,R}(T)(s) = A_{r} \sin \omega_{r,R}(T)s$$

(33)

where $A_{r} \in \mathbb{R}^{3}$ is the amplitude and $\omega_{r,R}(T) \in \mathbb{R}^{3}$ is the period of the antenna phase center fluctuation, and $\sin \omega_{r} = [\sin(\omega_{r1}), \sin(\omega_{r2}), \sin(\omega_{r3})]$.

5. Position error analysis for special trajectory errors

In this section, we describe the positioning errors in bistatic SAR images due to low-order polynomial trajectory errors and high-frequency sinusoidal trajectory errors.

We first define the angles$^{2}$ that are commonly used in describing bistatic geometries. These angles are illustrated in figure 1.

- $\theta_{h}$: bistatic angle between $\gamma_{T}(s)$ and $z$.
- $\psi_{g}$: grazing angle between the bistatic look-direction, $\hat{b}$, and the tangent plane of the ground.
- $\phi_{T,R}$: viewing angle of the transmitter/receiver.
- $\theta_{T,R}$: the angle between $\gamma_{T}(R)$ and $\Delta \gamma_{T,R}(s)$.

$^{2}$ All angles defined above are $s$ and $z$ dependent. We ignore this dependency for the sake of notational clarity.
• $\psi_{b,\perp}$: grazing angle between $v^T_b$ and the tangent plane of the ground.

We express (16) and (20) in terms of the angles described above as follows:

$$
\Delta z^E = \frac{1}{2 \cos \frac{\psi}{2} \cos \psi_b} \left[ e_1 \Delta \gamma_T^r (s) + e_2 \Delta \gamma_R^r (s) \right].
$$

(34)

$$
\Delta z^E = \frac{1}{\sqrt{v_b^T \cos \psi_{b,\perp}}} \left[ e_1 \Delta \gamma_T^r (s) + e_2 \Delta \gamma_R^r (s) \right]
$$

$$
+ \frac{1}{\sqrt{v_b^T \cos \psi_{b,\perp}}} \left[ e_1 \left| \gamma_T \cos \theta_T \left| \Delta \gamma_T^r (s) \right| \left| \frac{y_T (s) - z}{y_T (s) - z} \right| \right]
$$

$$
+ e_2 \left| \gamma_R \cos \theta_R \left| \Delta \gamma_R^r (s) \right| \right].
$$

(35)

where $\Xi$ and $\Xi^\perp$ are given by $^3$.

$$
\Xi (s, z) = 2 \cos \frac{\psi}{2} \left[ D\gamma (z) \hat{b} \right].
$$

(36)

$$
\Xi (s, z) = \frac{1}{\left| y_R (s) - z \right|} \left[ D\gamma (z) v^T_b \right].
$$

(37)

Note that $D\gamma (z)$ is an operator onto the tangent plane of the ground topography. $v^T_b$ in (36) and (38) denotes a function of the transverse antenna velocities given by

$$
v^T_b = \left| y_T \right| \sin \phi_T \left| \frac{y_T (s) - z}{y_T (s) - z} \right| \hat{T}^\perp + \left| y_R \right| \sin \phi_R \hat{r}^\perp.
$$

(38)

where $y_T (R) = \gamma_T^r (R) + \Delta \gamma_T^r (R)$.

We see from (35) and (36) that the positioning error induced by the transmitter and receiver trajectory errors are similar. To simplify our discussion, we assume that the transmitter trajectory has no error and consider only the positioning error induced by the receiver trajectory errors.

Assuming only the receiver trajectory error, (35) and (36) can be reduced to

$$
\Delta z^E = \frac{\Delta \gamma_R^r (s)}{2 \cos \frac{\psi}{2} \cos \psi_b},
$$

(39)

$$
\Delta z^E = \left[ y_R (s) - z \right] \Delta \gamma_R^r (s) + \left[ \gamma_R \cos \theta_R \right] \left| \Delta \gamma_R^r (s) \right|.
$$

(40)

The superscripts $r$ and $\perp$ in (40) and (41) present the components of a vector in the corresponding antenna look-direction and perpendicular to the antenna look-direction. We refer to these as the radial and transverse components, respectively.

We investigate the effects of low-frequency constant, linear and quadratic receiver trajectory errors and high-frequency sinusoidal trajectory error in the reconstructed images. Table 1 lists the positioning errors, $\Delta z^E$ and $\Delta z^E$, induced by these trajectory errors. The

$^3$ $\Xi$ can alternatively be expressed in terms of $\left| y_T (s) - z \right|$ as $1/\left| y_T (s) - z \right| D\gamma (z) v^T_b$.
positioning errors due to transmitter trajectory errors are similar to those in table 1 with the receiver motion error related parameters replaced with those of the transmitter.

We now summarize the implications of the results in table 1.

5.1. Analysis for low-frequency trajectory errors

5.1.1. Constant trajectory errors. We see that $\Delta z^R$ and $\Delta z^B$ involve antenna look-direction dependent terms. In wide-aperture imaging, transmitter and receiver look-directions vary significantly over the aperture. As a result, $\Delta z^R$ and $\Delta z^B$ vary with the slow-time. However, if the positioning errors are expressed in true geolocation coordinates, the slow-time dependency of the horizontal and vertical position errors is eliminated. Hence, constant trajectory errors result in a constant shift. This constant shift in positioning does not degrade the spatial integrity of the reconstructed images as long as the scene is small enough to ignore the dependency in shift. This is the case, for example, when a short aperture is used to image a small scene.

5.1.2. Linear trajectory errors. For a linear receiver trajectory error, $\Delta \gamma_R(s) = v_{r,RS}$, $\Delta z^R$ and $\Delta z^B$ are linearly dependent on the radial and transverse receiver velocity errors, respectively. We also note that in $\Delta z^B$, the first term in the summation is scaled by the range and hence is much larger than the second linear term and the radial positioning error, $\Delta z^B$. Thus, in this case, the transverse position error induced by the radial antenna velocity is much larger than the radial position error and the transverse position error induced by other sources.

For a short aperture, $\Delta z^R$ can be regarded as constant. Note that this shift is much larger than the transverse shift observed in the previous case under the same short aperture assumption due to the range term involved.

For a wide aperture and a large scene, the range term $|\gamma_R(s) - z|$ varies over the aperture and scene. As a result, the transverse position error $\Delta z^R$ varies with $s$ and $z$, which leads to spatially varying smearing artifacts in reconstructed images.

5.1.3. Quadratic trajectory errors. For the quadratic trajectory error $\Delta \gamma_R(s) = \frac{a_{r,RS}}{2} s^2$, both $\Delta z^R$ and $\Delta z^B$ vary quadratically with $s$, and are scaled by the radial and transverse...
acceleration errors, respectively. As in the previous case, the radial position error $\Delta \zeta$ is much smaller than the transverse position error $\Delta \zeta$. We see that the range dependent linear term in $\Delta \zeta$ leads to a spreading of the position error in the transverse look-direction resulting in smearing. This transverse smearing induced by the radial acceleration error $\mathbf{a}_r$ is much larger than the one described in the previous case.

For wide-aperture imaging, we also note that the dominant transverse smearing induced by the radial acceleration error may become nonlinear in $s$.

5.2. Analysis for sinusoidal trajectory errors

For sinusoidal receiver trajectory errors, both $\Delta \zeta$ and $\Delta \zeta$ vary sinusoidally. Due to the range term involved in the first term of $\Delta \zeta$, the transverse position error is much larger than the radial one, leading to a smearing in the bistatic transverse look-direction.

For a short aperture in which the changes in antenna look-directions are negligible, the position error manifests as a smearing artifact along the transverse direction in reconstructed images. For a wide aperture, the smearing resulting from the large transverse position error may vary in direction due to variations in the bistatic transverse look-direction.

In [1, 6] position errors due to low-frequency and high frequency trajectory errors were presented for a monostatic SAR traversing a straight linear trajectory. The analysis assumes that the images are reconstructed by a range-Doppler method. Our results for the low-frequency trajectory errors, listed in the first three rows of table 1, are qualitatively consistent with those reported in [1, 6]. However, for the high frequency sinusoidal trajectory errors, our result is different than the one in [1]. This difference is due to certain approximations and fast Fourier transform based processing involved in range-Doppler based image formation methods, such as the polar format algorithm [1].

6. Numerical simulations

This section presents simulation results to demonstrate the performance of our analysis. We consider two commonly used bistatic configurations: (i) bistatic constant-offset geometry; and
two-stationary bistatic SAR geometry. In bistatic constant off-set geometry, the transmitter and receiver traverse parallel linear trajectories at the same, constant velocity with a constant distance between the antennas [59, 60]. Figure 2 illustrates this geometry. The so-called ‘one-stationary’ bistatic SAR geometry involves a stationary transmitter, such as a TV or radio transmitter and a moving receiver [61, 62]. This configuration is illustrated in Figure 3.

In the numerical simulations, we consider the following four scenarios:

1. Bistatic constant-offset SAR configuration with low-frequency trajectory errors (constant along-track and cross-track offset errors, linear and quadratic errors) for a relatively short aperture.
2. Bistatic constant-offset SAR configuration with linear trajectory errors for a large aperture.
3. Bistatic constant-offset SAR configuration with linear trajectory errors for a large scene.
4. One-stationary bistatic SAR configuration with sinusoidal trajectory errors and a large circular receiver trajectory.

The system and waveform related parameters used in our simulations are close to those used in the GOTCHA experiment [63]. The transmitted waveform is a linear frequency modulated signal with a bandwidth of 124 MHz and a carrier frequency of 9.6 GHz. The antenna velocity is 69.4 m s\(^{-1}\) in all bistatic configurations.
The size and the discretization of the scene and the point target position are listed in table 2. Note that we considered two targets in the third set of simulations to show the \( z \) dependency of the positioning errors.

To verify our analysis, we compare the analytic and measured position errors in each scenario. The analytic radial and transverse position errors are obtained using (16) and (20) and the analytic vertical and horizontal position errors are obtained using (31) and (32). The measured position errors are obtained from the reconstructed images by estimating the position of the target. The estimation is based on the brightest pixel and subaperture processing where the data is divided into several patches in slow-time and the target location is determined for each slow-time value.

6.1. Bistatic constant-offset SAR with low-frequency trajectory errors

We assume the true transmitter and receiver trajectories are

\[
\gamma_T(s) = \gamma_{T,0} + v_{a,s} \quad \text{and} \quad \gamma_R(s) = \gamma_{R,0} + v_{s,s}
\]

where \( \gamma_{T,0} = [3.55e3 \ 0 \ 7.3e3] \ \text{m} \), \( \gamma_{R,0} = [3e3 \ 0 \ 0] \) and \( v_s = [69.4 \ 0 \ 0] \ \text{m s}^{-1} \). Thus, the receiver is on the right-hand-side and in front of the transmitter with \( D_1 = 3e3 \), \( D_2 = 0 \) and \( D_3 = 0 \).

6.1.1. Constant offset errors. We consider an along-track offset error and a cross-track offset error in the receiver trajectory corresponding to \( \Delta \gamma_{R,1} = [10, 0, 0] \ \text{m} \) and \( \Delta \gamma_{R,2} = [0, -10, 0] \ \text{m} \), respectively. Figure 4 shows the setup for this scenario with a cross-track offset error. The synthetic aperture length is 150 m for both the transmitter and receiver, as shown by the red and green dotted lines.

Figure 5(a) shows the reconstructed image for the bistatic constant-offset linear SAR without any trajectory errors. The target is well-focused as expected. The two trails
intersecting at the target position correspond to the bistatic look-direction (shown in red), \( \hat{\Xi} \), and the transverse bistatic look-direction (shown in white), \( \hat{\Xi} \), used in this setup.

Figures 5(b) and (c) show the reconstruction results for the cases of the along-track, \( \gamma_{R,1} \), and the cross-track, \( \gamma_{R,2} \), constant offset errors in the receiver. We see that the position error is simply a constant position shift in the bistatic transverse look-direction, \( \hat{\Xi} \), for the along-track offset error, as shown in figure 5(b). Similarly, it is a constant position shift in the bistatic look-direction, \( \hat{\Xi} \), for the cross-track offset error, as shown in figure 5(c).

The analytic and measured radial and transverse positioning errors for \( \gamma_{R,1} \) and \( \gamma_{R,2} \), are shown in figures 6(a) and (b), respectively. \( \Delta z^\Xi \) and \( \Delta z^\Xi \), remain almost constant throughout the synthetic aperture, which indicates a constant shift in target position in the reconstructed images, as shown in figures 5(b) and (c). This is consistent with our analysis for constant trajectory errors in section 5.1.1 when a relatively short aperture is used for imaging.

The analytic and measured vertical and horizontal positioning errors for \( \gamma_{R,1} \) and \( \gamma_{R,2} \), are shown by the dash-dotted lines in figures 6(a) and (b), respectively. Considering the
directions of $\hat{\Xi}$ and $\hat{\Xi}$ in this setup, as indicated in figure 5(a), the vertical positioning error, $\Delta z_2$ and the horizontal positioning error, $\Delta z_1$ are close to the radial and transverse positioning errors, respectively. The comparisons in the four diagrams shown in figures 6(a) and (b) verify this finding.

Note that the difference between the measured and analytic values may be due to the resolution limits imposed by the discretization of the scene.

6.1.2. Linear and quadratic receiving trajectory error. We consider linear and quadratic trajectory errors and assume that $v_e = [0, 0.3, -0.1] \text{ m s}^{-1}$ and $a_e = [0, 0.07, -0.07] \text{ m s}^{-2}$.
in the simulations. Figures 7(a) and (b) show the bistatic SAR images reconstructed for these two different low-frequency trajectory errors.

As compared to the reconstruction shown in figure 5(a), we see that there are constant position shifts in the reconstructed images for the case of linear trajectory errors, as shown in figure 7(a). For the case of the quadratic trajectory error, we see a smearing in the transverse antenna look-direction as shown in figure 7(b). These reconstruction results are consistent with our analysis summarized in sections 5.1.2 and 5.1.3 for a relatively short aperture imaging.

Figure 8(a) shows the analytic and measured radial and transverse target positioning errors, $\Delta z^R$ and $\Delta z^B$ as well as the analytic and measured vertical and horizontal target positioning errors, $\Delta z_2$ and $\Delta z_1$ for the cases of linear trajectory errors. The consistency of the analytic and measured values, as indicated by the black and blue curves, verifies our theoretical derivation. We see that the four curves remain almost flat throughout the relatively short aperture, another demonstration of constant shift error in target positioning as shown in figure 7(a).

As shown in figure 8(a), the transverse positioning error due to the antenna velocity error is much larger than the radial component, which can also be observed in the reconstructed image in figure 7(a). This is supported with our error analysis summarized section 5.1.2, where it is shown that this type of trajectory error results in a large position shift in the transverse look-direction.

Figure 8(b) shows the analytic and measured radial and transverse positioning errors and the vertical and horizontal positioning errors versus slow-time for the case of the quadratic antenna trajectory error. We see that the measured values match the analytic ones. Additionally, the linearity of the transverse position error with respect to slow-time is consistent with our analysis in section 5.1.3. This slow-time dependence results in a smearing in the transverse look-direction, as shown in figure 7(b).
The quadratic behavior of the transverse positioning error shown in table 1 is not observable in neither figure 7(b) nor figure 8(b), because the quadratic term is much smaller than the linear term. The radial positioning error is negligible due to the small radial acceleration error and the short synthetic aperture.

Due to the squint-look\(^4\) in the current SAR configuration, as shown in figure 4, where the bistatic look-direction \(\Xi\) and transverse look-direction \(\hat{\Xi}\) are not along the vertical and horizontal directions, we see that the target positioning error has both evident vertical and horizontal components as expected, as shown in figure 8. For instance, the dominant linear transverse smearing due to a quadratic trajectory error leads to linear position errors in both vertical and horizontal directions, as shown in figure 8(b). Note that as shown in figures 8(a) and (b), the horizontal positioning error is much close to the transverse positioning error. This is due to the fact that the bistatic look direction \(\Xi\) is almost parallel to the vertical axis in the current setup and accordingly the \(\Delta x^\Xi\) contributes more to the horizontal positioning error.

6.2. Wide-aperture imaging using bistatic constant-offset SAR with linear trajectory errors

In this section, we used the same bistatic SAR configuration as in section 6.1, but with the aperture length increased from 150 m to 8e3 m. As a result, the scene is discretized into a much denser grid, as listed in table 2, to obtain more accurate target positioning error. We assumed the same trajectory error, \(\nu_{e,R} = [0, 0.3, 0.1] \text{ m s}^{-1}\), as in section 6.1.

Figure 9(a) shows the reconstructed bistatic SAR image. The red dot in figure 9(a) indicates the correct target position. Figure 9(b) presents the analytic and measured radial and transverse, vertical and horizontal position errors of the target, which indicates a good match of the analytic and measured values.

\(^4\) At the central position over the aperture, the look-direction of the radar is not perpendicular to the direction of the antenna velocity [34].
Comparing figures 9(a) with 7(a), we see that in the case of wide-aperture imaging, the target positioning errors in the reconstructed image induced by the linear antenna trajectory errors can no longer be regarded as constant shifts. This is due to the large changes in the bistatic look-direction and transverse look-direction as well as large range variations between the antennas and the target over the long aperture, as analyzed in section 5.1.2.

Figure 10. The reconstructed bistatic SAR images for (a) one target located at [128, 128]th pixel and (b) the other target located at [3968, 3968]th pixel. The imaging setup is as shown in figure 4, however with a much larger scene. A linear receiver trajectory error, \( v_{t,R} = [0.3 \ -0.1] \text{ m s}^{-1} \) was assumed. The red dot denote the correct target position.

Figure 11. The analytic and measured radial and transverse, vertical and horizontal position errors of (a) the target located at [128, 128]th pixel and (b) the target located at [3968, 3968]th pixel for a linear receiving trajectory error with \( v_{t,R} = [0.3 \ -0.1] \text{ m s}^{-1} \). The black curves denote the analytic values and the blue curves denote the measured values.
As shown in figure 9(a), there is an obvious smearing occurring mainly in the horizontal direction. This smearing is also indicated by the vertical and horizontal positioning error plots shown in figure 9(b), in which the vertical positioning error is almost constant and the horizontal positioning error varies over the aperture.

6.3. Large scene imaging by bistatic constant-offset SAR with linear trajectory errors

In this section, we used the same bistatic SAR configuration and the aperture length of 150 m as in section 6.1, but enlarged the size of the scene by a factor of 256, as listed in table 2. The linear receiver trajectory error, \( v_{r,R} = [0, 0.3, -0.1] \) m s\(^{-1}\), is the same as in sections 6.1 and 6.2.

Figure 10 shows the reconstructed images of the two targets, as shown in figures 10(a) and (b), respectively. Figure 11 shows the analytic and measured radial and transverse positioning errors and the vertical and horizontal positioning errors versus slow-time for the two targets.

We see that the linear receiver trajectory error induces constant transverse shifts in both targets, as we analyzed in section 5.1.2. However, the shifts are different for each target, as can be observed from figures 10(a) and (b). This position dependent shift is also indicated by the position error plots shown in figures 11(a) and (b). This verifies the \( z \) dependency of the positioning error that we derived in section 5.1.2.

6.4. One-Stationary bistatic SAR with sinusoidal antenna trajectory errors

Figure 12 shows the setup used in this section. We assume that the receiver trajectory is \( \gamma(s) = [7.1e3 + \cos(v_t s/R), 7.1e3 + \sin(v_t s/R), 7.3e3] \) where \( v_t = 69.4 \) m s\(^{-1}\) and \( R = 7.1e3 \) denotes the radius of the circular trajectory, as shown by the dashed line in figure 12. The fixed transmitter is assumed to be located at \([14.6e3, 14.6e3, 0.4e3]\), as shown by the 'diamond' in figure 12. We consider a sinusoidal receiver trajectory.
error as shown by the solid line in figure 12 and assume $\Delta y = [0.5 \cos(s/5) - 0.5 \sin(s/5) \ 0]$ m. The synthetic aperture length corresponds to a rotation angle of $\pi/4$ and a time of 80.3505 s. The initial antenna position corresponds to $-\pi/8$, as show in figure 12.

Figure 13(a) shows the reconstructed image without any trajectory errors and with (b) a sinusoidal trajectory error $\Delta y(s) = [0.5 \cos(s/5) - 0.5 \sin(s/5) \ 0]$ m for the one-stationary bistatic configuration shown in figure 12. The red dot in figure 13(b) denotes the true target position. (c) figure 13(a) in logarithm scale. (d) figure 13(b) in logarithmic scale.

Figure 13. The reconstructed one-stationary bistatic SAR images (a) without any trajectory errors and with (b) a sinusoidal trajectory error $\Delta y(s) = [0.5 \cos(s/5) - 0.5 \sin(s/5) \ 0]$ m for the one-stationary bistatic configuration shown in figure 12. The red dot in figure 13(b) denotes the true target position. (c) figure 13(a) in logarithm scale. (d) figure 13(b) in logarithmic scale.

error as shown by the solid line in figure 12 and assume $\Delta y(s) = [0.5 \cos(s/5) - 0.5 \sin(s/5) \ 0]$ m. The synthetic aperture length corresponds to a rotation angle of $\pi/4$ and a time of 80.3505 s. The initial antenna position corresponds to $-\pi/8$, as show in figure 12.

Figure 13(a) shows the reconstructed image without any antenna trajectory errors for the one-stationary bistatic SAR configuration shown in figure 12. We see that the target is reconstructed at the correct position with good contrast. Figure 13(b) shows the reconstructed image when the receiver trajectory experiences sinusoidal fluctuations. In this case, the target energy is not focused at the true target position, but instead, spreads leading to the smearing artifact in the bistatic transverse look-direction. Figures 13(c) and (d) show the images in figures 13(a) and (b) in logarithmic scale to enhance the artifacts and target image. These images show the variation in the bistatic look-direction and transverse look-direction during the aperture.

Figure 14 shows the analytic and measured radial positioning errors and transverse positioning errors as well as the analytic and measured vertical positioning errors and
horizontal positioning errors versus the slow-time. The measured error matches the analytical ones well throughout the aperture.

Figures 13(b) and (d) show that the smearing is not along a single transverse look-direction, but along a set of adjacent transverse look-directions. This smearing pattern can be explained by the large transverse positioning errors as compared to the radial errors, as shown in figure 14 and the large variations in the bistatic look-directions throughout the large circular aperture, as indicated by the red arrows in figure 13(c). It can be predicted that the vertical and horizontal positioning errors, which are components of the smearing in vertical and horizontal directions, vary significantly during the aperture and follow different sinusoidal curves due to the sinusoidal transverse positioning error. The curves presented in figure 14 verify this.

As shown in figure 13(c), the bistatic look-directions, \( \Xi \) during the data collection of the current one-stationary bistatic SAR setup are located near the horizontal axis. Thus, the horizontal positioning error, \( \Delta z_1 \) mainly depends on the radial positioning error. The vertical positioning error, \( \Delta z_2 \) is close to the transverse positioning error, as shown in figure 14. However, since \( \Delta z_2^\Xi \) is too small, the large transverse positioning error becomes dominant and \( \Delta z_1^\Xi \) also follows a sinusoid similar to the transverse positioning error, as shown in figure 14.

### 7. Conclusion

We presented a theory to analyze the positioning errors in bistatic SAR images due to antenna trajectory errors. Our results are applicable to arbitrary trajectory errors in transmitting and receiving antennas, arbitrary imaging geometries, and monostatic and bistatic configurations.

We used microlocal analysis to derive the position errors in the radar look-direction and transverse look-direction. The analysis provides an explicit quantitative relationship between the trajectory error and the position errors in bistatic SAR images. Our analysis shows that the radial component of the trajectory error contributes more to the position errors than the
transverse component of the trajectory error. This is consistent with the fundamental principle of wideband SAR, which is designed to acquire high-resolution in the radial direction.

We studied a general bistatic SAR imaging in the presence of low-frequency trajectory errors including constant, linear and quadratic errors, and the high frequency sinusoidal trajectory errors. The results show that the constant trajectory errors lead to constant position shifts in the reconstructed SAR images. The linear trajectory error leads to a much larger shift in the transverse direction as compared to that of radial direction. The quadratic trajectory error leads to a larger transverse smearing as compared to the small radial position error. The high frequency sinusoidal trajectory error induces significant smearing in the bistatic transverse look-direction. We also analyze the characteristics of the positioning errors in wide-aperture and short-aperture imaging as well as large-scene imaging.

We present numerical simulations in different scenarios to demonstrate our analysis, including bistatic constant-offset SAR configurations and one-stationary bistatic SAR configuration, conventional straight linear trajectory and non-traditional circular trajectories, low-frequency trajectory errors and sinusoidal trajectory errors. The position errors obtained from the reconstructions are consistent with the analytic ones provided by the theory.

The antenna trajectory error analysis presented in this paper is useful for evaluating the effect of different types of trajectory errors in reconstructed SAR images. Furthermore, the explicit relationship between the positioning errors and antenna trajectory errors can be used to determine upper limits on the antenna trajectory errors to constrain positioning errors within an acceptable range.

Our study can be utilized in developing model-based autofocus methods for SAR image formation. Depending on imaging geometry, appropriate error models from our study can be incorporated into image formation or into image quality optimization to form focused images. Our predictive models can be used to estimate antenna trajectory errors from reconstructed images and next these estimates can be used to form focused images. Alternatively, the error models can be incorporated into the reconstruction and antenna errors and a focused image can be estimated simultaneously based on an image quality figure of merit. We leave for the future the task of developing autofocus methods based on our motion error analysis.

The analysis methodology presented in this paper can be used to analyze the effect of the trajectory errors in other SAR modalities, such as ultranarrowband continuous-wave (CW) SAR [43, 64, 65], passive SAR including SAR hitchhiker [45] and Doppler hitchhiker [42, 44].

Although our analysis was developed in a deterministic setting, it is can be extended to include random trajectory errors and additive noise [52].

Finally, the approach used in our study can be extended to analyze the effect of other factors, such as the variation in the electromagnetic wave speed.

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Appendix A. Derivation of (16)

We make the Taylor series approximation of

$$R\left(\gamma_T(s) - e_1 \Delta \gamma_T(s), \gamma_R(s) - e_2 \Delta \gamma_R(s), \mathbf{z} + \Delta \mathbf{z}\right)$$

around $e_1 = 0$ and $e_2 = 0$ and have

$$R\left(\gamma_T(s), \gamma_R(s), \mathbf{z}\right) 
\approx R\left(\gamma_T(s) - e_1 \Delta \gamma_T(s), \gamma_R(s) - e_2 \Delta \gamma_R(s), \mathbf{z}\right) 
- e_1 \partial_{e_1} R\left(\gamma_T(s), \gamma_R(s), \mathbf{z}\right)\bigg|_{e_1 = 0} 
- e_2 \partial_{e_2} R\left(\gamma_T(s), \gamma_R(s), \mathbf{z}\right)\bigg|_{e_2 = 0} 
+ \nabla_{s} R\left(\gamma_T(s), \gamma_R(s), \mathbf{z}\right) \cdot \Delta \mathbf{z}. \tag{A.1}$$

Substituting (A.1) into (10), we obtain (14). Using (2), we have

$$\partial_{e_1} R\left(\gamma_T(s) - e_1 \Delta \gamma_T(s), \gamma_R(s), \mathbf{z}\right) 
= \frac{d\gamma_T(s) - e_1 \Delta \gamma_T(s) - \mathbf{z}}{de_1} 
= \left(\gamma_T(s) - e_1 \Delta \gamma_T(s) - \mathbf{z}\right) \cdot \left(-\Delta \gamma_T(s)\right). \tag{A.2}$$

Similarly, we have

$$\partial_{e_2} R\left(\gamma_T(s), \gamma_R(s) - e_2 \Delta \gamma_R(s), \mathbf{z}\right) 
= \frac{d\gamma_R(s) - e_2 \Delta \gamma_R(s) - \mathbf{z}}{de_2} 
= \left(\gamma_R(s) - e_2 \Delta \gamma_R(s) - \mathbf{z}\right) \cdot \left(-\Delta \gamma_R(s)\right). \tag{A.3}$$

Using (2) and referring to [48], we have

$$\nabla_{s} R\left(\gamma_T(s), \gamma_R(s), \mathbf{z}\right) = -\Xi(s, \mathbf{z}) 
\quad = -D_{\Psi}\left[\gamma_T(s) - \mathbf{z}\right] + \left(\gamma_R(s) - \mathbf{z}\right). \tag{A.4}$$

Substituting (B.2), (A.3) and (A.4) back into (14), we obtain

$$\Delta \mathbf{z} \cdot \Xi(s, \mathbf{z}) = \left[e_1 \Delta \gamma_T(s) \cdot \left(\gamma_T(s) - \mathbf{z}\right) + e_2 \Delta \gamma_R(s) \cdot \left(\gamma_R(s) - \mathbf{z}\right)\right]. \tag{A.5}$$
Appendix B. Derivation of (20)

We make the following Taylor series approximation of 
\[ f(\gamma_T(s) - e_1 \Delta \gamma_T(s), \gamma_R(s) - e_2 \Delta \gamma_R(s), z + \Delta z) \]
around \( \epsilon_1 = 0 \) and \( \epsilon_2 = 0 \) and have
\[
\begin{align*}
&f(\gamma_T(s) - e_1 \Delta \gamma_T(s), \gamma_R(s) - e_2 \Delta \gamma_R(s), z + \Delta z) \\
&\approx f(\gamma_T(s), \gamma_R(s), z) \\
&- e_1 \partial_{\epsilon_1} f(\gamma_T(s) - e_1 \Delta \gamma_T(s), \gamma_R(s), z) \bigg|_{\epsilon_1=0} \\
&- e_2 \partial_{\epsilon_2} f(\gamma_T(s), \gamma_R(s) - e_2 \Delta \gamma_R(s), z) \bigg|_{\epsilon_2=0} \\
&+ V_z f(\gamma_T(s), \gamma_R(s), z) \cdot \Delta z.
\end{align*}
\]
(B.1)

Substituting (B.1) into (11), we obtain (15). Using (7), we have
\[
\partial_\epsilon f(\gamma_T(s) - e_1 \Delta \gamma_T(s), \gamma_R(s), z)
= \frac{d}{d\epsilon_1} \left[ \left( \gamma_T(s) - e_1 \Delta \gamma_T(s) \right) \cdot \left( \gamma_T(s) - e_1 \Delta \gamma_T(s) - z \right) \right]
= \omega \left[ \left( \gamma_T(s) - e_1 \Delta \gamma_T(s) \right) \cdot \left( \gamma_T(s) - e_1 \Delta \gamma_T(s) - z \right) \right]
\left( \gamma_T(s) - e_1 \Delta \gamma_T(s) \right) \cdot \left( \gamma_T(s) - e_1 \Delta \gamma_T(s) - z \right)
= \partial_{\epsilon_1} \left[ \left( \gamma_T(s) - e_1 \Delta \gamma_T(s) - z \right) \right]
= \left[ \gamma_T(s) - e_1 \Delta \gamma_T(s) - z \right] \\
\left[ \gamma_T(s) - e_1 \Delta \gamma_T(s) - z \right]
= -\Delta \gamma_T(s) \\
\left( \gamma_T(s) - e_1 \Delta \gamma_T(s) - z \right)
\left[ \gamma_T(s) - e_1 \Delta \gamma_T(s) - z \right]
\times \left[ \Delta \gamma_T(s) \right]
= \left. \left( \gamma_T(s) - e_1 \Delta \gamma_T(s) - z \right) \cdot \Delta \gamma_T(s) \right].
\]
(B.2)

where
\[
\begin{align*}
\left( \gamma_T(s) + e_1 \Delta \gamma_T(s) - z \right)
&= \left( \gamma_T(s) - e_1 \Delta \gamma_T(s) - z \right)
\times \left( \gamma_T(s) - e_1 \Delta \gamma_T(s) - z \right)
\times \left( \gamma_T(s) - e_1 \Delta \gamma_T(s) - z \right)
\times \left. \left( \gamma_T(s) - e_1 \Delta \gamma_T(s) - z \right) \cdot \Delta \gamma_T(s) \right]
= \left. \left( \gamma_T(s) - e_1 \Delta \gamma_T(s) - z \right) \cdot \Delta \gamma_T(s) \right]
\end{align*}
\]
Substituting (B.3) back into (B.2), we obtain

\[
\partial_{c_{3}}f(\gamma_{r}(s) + \epsilon_{1}\Delta\gamma_{r}(s), \gamma_{r}(s), z) \bigg|_{z=0} = -\alpha \left[ \frac{\Delta\gamma_{r}^{\perp}(s)}{\gamma_{r}(s) - z} \right],
\]

(B.4)

and similarly we have

\[
\partial_{c_{3}}f(\gamma_{r}(s), \gamma_{r}(s) + \epsilon_{2}\Delta\gamma_{r}(s), z) \bigg|_{z=0} = -\alpha \left[ \frac{\Delta\gamma_{r}^{\perp}(s)}{\gamma_{r}(s) - z} \right],
\]

(B.5)

where \(\Delta\gamma_{r}^{\perp}(s)\) and \(\Delta\gamma_{r}^{\perp}(s)\) are as defined in (23).

Considering the fact that \(V_{c_{2}}f = \alpha V_{r}(\partial_{R}) = \omega \partial_{(\alpha R)} = -\omega \partial_{\Xi}\), we have

\[
V_{c_{2}}f(\gamma_{r}(s), \gamma_{r}(s), z) = -\alpha \frac{\partial\Xi(s, z)}{\partial s}.
\]

(B.6)

Let \(\Xi(s, z) = \partial\Xi(s, z)/\partial s\). Substituting (B.6), (B.4) and (B.5) into (B.1), we obtain (20).

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